GAIT GENERATION FOR A HOPPING ROBOT VIA ITERATIVE LEARNING CONTROL BASED ON VARIATIONAL SYMMETRY

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Abstract: This paper proposes a novel framework to generate optimal gait trajectories for a one-legged hopping robot via iterative learning control. This method generates gait trajectories which are solutions of a class of optimal control problems without using precise knowledge of the plant model. It is expected to produce natural gait movements such as that of a passive walker. Some numerical examples demonstrate the effectiveness of the proposed method.

Keywords: Nonlinear control, Hamiltonian systems, Iterative learning control, Legged locomotion, Gait generation

1. INTRODUCTION

Passive dynamic walking originally studied by McGeer (McGeer, 1990) inspires many researchers to work on a gait generation problem for walking robots. They try to design more natural and less energy consuming gait trajectories than those produced by conventional walking control such as ZMP based control. Behavior analysis of passive walkers were investigated, e.g, in (Osuka and Kirihara, 2000; Sano *et al.*, 2003). There are some results on gait generation based on passive dynamic walking (Goswami *et al.*, 1997; Spong, 1999; Asano *et al.*, 2004) by designing appropriate feedback control systems such that the closed loop systems behave like passive walkers. In particular, a hopping robot modelled in (Ahmadi and Buehler, 1997) has a hopping gait for which the input signal coincides with zero, that is, this robot can be regarded as a passive walker walking on a horizontal plane. Furthermore, an adaptive control system for this robot to achieve a walking gait with zero input was proposed in the authors' former result (Hyon and Emura, 2004).

The objective of this paper is to generate optimal walking gait trajectories for a hopping robot via iterative learning without using precise knowledge of the plant model. To this end, we formulate an optimal control type cost function and try to find a control input minimizing it by iterative learning technique based on *variational symmetry* of Hamiltonian control systems (Fujimoto and Sugie, 2003), which can solve a class of optimal control problems by iteration of experiments. For this purpose, two novel techniques with respect to iterative learning control are proposed: One is a technique to take the time derivatives of the output signal into account in the iterative learning control by employing a pseudo adjoint of the time derivative operator. The other is a cost function to achieve time symmetric gait trajectories to guarantee stable walking without a fall. Furthermore, the propose learning scheme is applied to the hopping robot in (Ahmadi and Buehler, 1997) and the corresponding numerical simulations demonstrate its advantage.

2. ITERATIVE LEARNING CONTROL BASED ON VARIATIONAL SYMMETRY

This section refers to the iterative learning control (ILC) method based on variational symmetry in (Fujimoto and Sugie, 2003) briefly.

2.1 Variational symmetry of Hamiltonian systems

Consider a Hamiltonian system with dissipation and a controlled Hamiltonian $H(x,u,t)$ described by

$$\Sigma : \begin{cases} \dot{x} = (J-R)\dfrac{\partial H(x,u,t)}{\partial x}^{\mathrm{T}}, \quad x(t^0) = x^0 \\ y = -\dfrac{\partial H(x,u,t)}{\partial u}^{\mathrm{T}} \end{cases} \tag{1}$$

Here $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$ and $y(t) \in \mathbb{R}^r$ describe the state, the input and the output, respectively. The structure matrix $J \in \mathbb{R}^{n\times n}$ and the dissipation matrix $R \in \mathbb{R}^{n\times n}$ are skew-symmetric and symmetric positive semi-definite, respectively. The matrix R represents dissipative elements. In this paper the Hamiltonian system in (1) is written as $y = \Sigma(u)$. For this system, the following theorem holds. This property is called *variational symmetry* of Hamiltonian control systems.

Theorem 1. (Fujimoto and Sugie, 2003) Consider the Hamiltonian system in (1). Suppose that J and R are constant and that there exists a nonsingular matrix $T \in \mathbb{R}^{n\times n}$ satisfying

$$\begin{aligned} J &= -TJT^{-1} \\ R &= TRT^{-1} \end{aligned} \tag{2}$$

$$\frac{\partial^2 H(x,u,t)}{\partial (x,u)^2} = \begin{pmatrix} T & 0 \\ 0 & I \end{pmatrix} \frac{\partial^2 H(x,u,t)}{\partial (x,u)^2} \begin{pmatrix} T^{-1} & 0 \\ 0 & I \end{pmatrix}$$

Suppose moreover that $J-R$ is nonsingular. Then the variational system $\mathrm{d}\Sigma$ of Σ and its adjoint $(\mathrm{d}\Sigma)^*$ of Σ have almost the same state-space realizations.

Remark 2. Suppose the Hessian of the Hamiltonian with respect to (x,u) is satisfying

$$\frac{\partial^2 H(x,u,t)}{\partial (x,u)^2}(t-t^0) = \frac{\partial^2 H(x,u,t)}{\partial (x,u)^2}(t^1-t), \quad {}^\forall t \in [t^0, t^1] \tag{3}$$

Then under the appropriate initial condition of Σ and when v is small, (4) holds.

$$(\mathrm{d}\Sigma(u))^*(v) \approx \mathcal{R} \circ (\Sigma(u + \mathcal{R}(v)) - \Sigma(u)) \tag{4}$$

where $\mathcal{R}$ is a time-reversal operator defined by $\mathcal{R}(u)(t-t^0) = u(t^1-t)$ for $\forall t \in [t^0, t^1]$

Equation (4) implies that we can calculate the input-output mapping of the adjoint by only using the input-output data of the original system.

2.2 Optimal control via iterative learning

Let us consider the system Σ in (1) and a cost function $\Gamma : L_2^m[t^0,t^1] \times L_2^r[t^0,t^1] \to \mathbb{R}$ as follows

$$\begin{aligned} \Gamma(u,y) = \frac{1}{2}\int_{t^0}^{t^1} \Big(& u(t)^{\mathrm{T}}\Lambda_u u(t) \\ & + (y(t)-y^d(t))^{\mathrm{T}}\Lambda_y(y(t)-y^d(t))\Big)\,\mathrm{d}t \end{aligned} \tag{5}$$

where $y^d \in L_2^r[t^0,t^1]$ represents a desired output and $\Lambda_u \in \mathbb{R}^{m\times m}$ and $\Lambda_y \in \mathbb{R}^{r\times r}$ are positive definite matrices. The objective is to find the optimal input minimizing the cost function $\Gamma(u,y)$. Note that the Fréchet derivative of Γ is $\mathrm{d}\Gamma(u,y)$. It follows from well-known Riesz's representation theorem that there exists an operator $\Gamma'(u,y)$ such that

$$\begin{aligned} \mathrm{d}(\Gamma(u,y)) &= \mathrm{d}\Gamma(u,y)(\mathrm{d}u, \mathrm{d}y) \\ &= \langle \Gamma'(u,y), (\mathrm{d}u, \mathrm{d}y)\rangle_{L_2} \end{aligned} \tag{6}$$

Here we can calculate

$$\begin{aligned} \mathrm{d}(\Gamma(u,y)) &= \langle (\Lambda_u u, \Lambda_y(y-y^d)), (\mathrm{d}u, \mathrm{d}y)\rangle_{L_2} \\ &= \langle \Lambda_u u, \mathrm{d}u\rangle_{L_2} + \langle \Lambda_y(y-y^d), \mathrm{d}\Sigma(u)(\mathrm{d}u)\rangle_{L_2} \\ &= \langle \Lambda_u u + (\mathrm{d}\Sigma(u))^*\Lambda_y(y-y^d), \mathrm{d}u\rangle_{L_2} \end{aligned} \tag{7}$$

Therefore the steepest descent method implies that we should change the input u such that

$$\mathrm{d}u = -K\big(\Lambda_u u + (\mathrm{d}\Sigma(u))^*\Lambda_y(y-y^d)\big) \tag{8}$$

where K is an appropriate positive gain. Hence the iteration law should be taken as

$$u_{(i+1)} = u_{(i)} - K_{(i)}\big(\Lambda_u u_{(i)} + (\mathrm{d}\Sigma(u))^*\Lambda_y(y_{(i)} - y^d)\big) \tag{9}$$

Here i denotes the i-th iteration in laboratory experiment.

Suppose (3) holds, then ILC law based on variational symmetry is given by

$$u_{(2i-1)} = u_{(2i-2)} + \mathcal{R}(\Lambda_y(y_{(2i-2)} - y^d)) \quad (10)$$

$$u_{(2i)} = u_{(2i-2)} - K_{(2i-2)}\big(\Lambda_u u_{(2i-2)} + \mathcal{R}(y_{(2i-1)} - y_{(2i-2)})\big) \quad (11)$$

provided that the initial input $u_{(0)}$ is equivalent to zero.

This pair of iteration laws (10) and (11) implies that this learning procedure needs two steps laboratory experiments. In the (2i-1)-th iteration, we can get the output signal of $\Sigma(u+\mathcal{R}(v))$ in (4) and then can calculate the input and output signals of $(\mathrm{d}\Sigma(u))^*$ from (4). Input for the 2i-th iteration is generated by (9) with these signals.

3. EXTENSION OF ILC FOR TIME DERIVATIVES

Let us recall that there is a constraint with respect to cost functions in the iterative learning control method in (Fujimoto and Sugie, 2003). For the system Σ in (1), the output y is uniquely defined by the definition of the input u. The possible choice of the optimal control type cost function used in iterative learning control is a functional of u and y, and it is not possible to choose a functional of $\dot{y}$ the time derivative of the output. However, the signal $\dot{y}$ often plays an important role in control systems and, particularly, it is important to check the behavior of $\dot{y}$ for the gait trajectory generation problem. In this section, we extend the iterative learning control method referred in the previous section to take the time derivative $\dot{y}$ into account.

Let us consider the Hamiltonian system in (1) and suppose that the following assumption holds.

Assumption 3. Following conditions always hold $\mathrm{d}y(t^0) = 0$ and $\mathrm{d}y(t^1) = 0$

In iterative learning control, it is assumed that all the initial conditions are same in each laboratory experiment in general. Therefore the condition $\mathrm{d}y(t^0) = 0$ always holds. But the other one $\mathrm{d}y(t^1) = 0$ does not always hold. In order to let the latter condition $\mathrm{d}y(t^1) = 0$ hold approximately, we can employ an optimal control type cost function such as $\int_{t^1-\epsilon}^{t^1} \|y(t) - y^d(t)\|^2 \mathrm{d}t$ with a small constant $\epsilon > 0$ as in (Fujimoto *et al.*, 2003).

3.1 Pseudo adjoint of the time derivative operator

Here we investigate a pseudo adjoint of the time derivative operator to take account of the time derivative of the output signal $\dot{y}$ in the iterative learning control procedure.

Consider a differentiable signal $\xi \in L_2[t^0, t^1]$ and an operator $D(\cdot)$ which maps the signal $\xi(t)$ into its time derivative is defined as the time derivative operator.

$$D(\xi)(t) := \frac{\mathrm{d}\xi(t)}{\mathrm{d}t} \quad (12)$$

Let us provide the following lemma to define the pseudo adjoint of the time derivative operator.

Lemma 4. Consider the signal $\xi(t)$ defined above and another differentiable signal $\eta \in L_2[t^0, t^1]$. Suppose that the signal $\xi(t)$ satisfies the following condition

$$\xi(t^0) = \xi(t^1) = 0. \quad (13)$$

Then the following equation holds.

$$\langle \eta, D(\xi) \rangle_{L_2} = \langle -D(\eta), \xi \rangle_{L_2} \quad (14)$$

Proof Consider the inner product of η and $D(\xi)$. Let us calculate that

$$\langle \eta, D(\xi) \rangle_{L_2} = \int_{t^0}^{t^1} \eta(t)^{\mathrm{T}} \frac{\mathrm{d}\xi(t)}{\mathrm{d}t}\, \mathrm{d}t = \Big[\eta(t)^{\mathrm{T}}\xi(t)\Big]_{t^0}^{t^1} - \int_{t^0}^{t^1} \frac{\mathrm{d}\eta(t)}{\mathrm{d}t}^{\mathrm{T}} \xi(t)\, \mathrm{d}t \quad (15)$$

Here $\xi(t)$ satisfies the condition (13), therefore $\Big[\eta(t)^{\mathrm{T}}\xi(t)\Big]_{t^0}^{t^1} = 0$ holds and down to

$$\langle \eta, D(\xi) \rangle_{L_2} = -\int_{t^0}^{t^1} \frac{\mathrm{d}\eta(t)}{\mathrm{d}t}^{\mathrm{T}} \xi(t)\, \mathrm{d}t = \langle -D(\eta), \xi \rangle_{L_2} \quad (16)$$

Then (16) implies (14). □

This lemma implies

$$D^* = -D$$

for a certain class of input signals.

3.2 Application to iterative learning control

Here we take the following cost function to illustrate the proposed method.

$$\Gamma(\dot{y}) = \frac{1}{2}\int_{t^0}^{t^1} \Big((\dot{y}(t) - \dot{y}^d(t))^{\mathrm{T}} \Lambda_{\dot{y}} (\dot{y}(t) - \dot{y}^d(t))\Big)\, \mathrm{d}t \quad (17)$$

Here $\dot{y}^d$ is a differentiable signal as a desired velocity and satisfies $\dot{y}^d \in L_2[t^0, t^1]$. Suppose that the output y holds Assumption 3. Then we have

$$\mathrm{d}(\Gamma(\dot{y})) = \langle \Lambda_{\dot{y}}(\dot{y} - \dot{y}^d),\ \mathrm{d}\dot{y} \rangle_{L_2} \quad (18)$$

The authors' former result (Fujimoto and Sugie, 2003) can not directly apply to this cost function

(17) because it contains $\dot{y}$. Here let us rewrite $\dot{y}$ as $\dot{y} = D(y)$ with the time derivative operator $D(\cdot)$ defined by Equation (12). Then we have

$$\mathrm{d}\dot{y} = \mathrm{d}D(y)(\mathrm{d}y). \tag{19}$$

Note that the time derivative operator is linear, we obtain $\mathrm{d}\dot{y} = D(\mathrm{d}y)$. Using Assumption 3 and (14), we can down to

$$\begin{aligned}\mathrm{d}(\Gamma(\dot{y})) &= \langle \Lambda_{\dot{y}}(\dot{y} - \dot{y}^d), D(\mathrm{d}y)\rangle_{L_2} \\ &= \langle -D\Big(\Lambda_{\dot{y}}(\dot{y} - \dot{y}^d)\Big),\ \mathrm{d}y\rangle_{L_2} \\ &= \langle (\mathrm{d}\Sigma(u))^* \Big(-D(\Lambda_{\dot{y}}(\dot{y} - \dot{y}^d))\Big),\ \mathrm{d}u\rangle_{L_2}\end{aligned} \tag{20}$$

As we mentioned above, this proposed method allows one to apply the ILC to cost functions of state variables which do not appear in the output y.

4. OPTIMAL GAIT GENERATION

In this section, a cost function to generate a symmetric gait is proposed and the method in the previous section is applied to it.

4.1 Description of the plant

Let us consider a passive hopping robot in (Ahmadi and Buehler, 1997; Hyon and Emura, 2004) depicted in Figure 1.

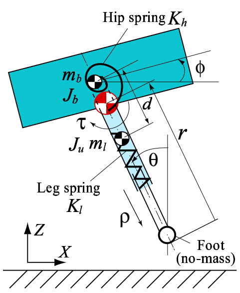

Fig. 1. Description of the plant

Here the body and the leg have mass m_b and m_l and moment of inertia J_b and J_u. The mass of the leg is located below the hip joint, with distance d. Let us define the equivalent leg inertia J_l as follows

$$J_l = J_u + \frac{m_b m_l}{m_b + m_l} d^2. \tag{21}$$

Let us also define the control force of the leg ρ and the control torque of the hip joint τ. Table 1 shows the physical parameters. In (Hyon and Emura, 2004), See (Hyon and Emura, 2004) for more detail.

Here the stance time represents the time interval during the stance phase and the flight time is defined in a similar way. Furthermore, we suppose the following assumption.

Table 1. Parameters

notation	Meaning	Unit
r_0	natural leg length	m
m	total mass	kg
g	gravity acceleration	m/s^2
T_s	stance time	s
T_f	flight time	s

Assumption 5. The foot does not bounce back nor slip on the ground (inelastic impulsive impact).

A notion of phases is introduced: the stance phase and the flight phase. When the leg touches the ground the robot is said to be in the stance phase, and when the leg is above the ground it is said to be in the flight phase. The robot moves between these two phases alternately. see Figure 2.

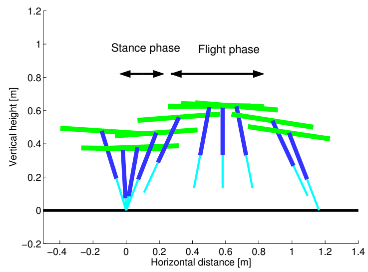

Fig. 2. Subsequent one step of hopping(The robot moves from left to right.)

In the stance phase, let us define the generalized coordinate q as $q := (r, \theta, \phi)^{\mathrm{T}} \in \mathbb{R} \times \mathbb{S}^1 \times \mathbb{S}^1$, the generalized momentum p as $p := (p_r, p_\theta, p_\phi)^{\mathrm{T}} \in \mathbb{R}^3$, input u as $u := (\rho, \tau)^{\mathrm{T}} \in \mathbb{R}^2$ and the inertia matrix as $M(q) \in \mathbb{R}^{3\times 3}$. Then, the dynamics is described as a Hamiltonian system in (1) with the Hamiltonian $H(q, p, u)$ represented as

$$\begin{aligned} H(q,p,u) &= \frac{1}{2}p^{\mathrm{T}} M(q)^{-1} p - mgr(1-\cos\theta) + \\ &+ \frac{1}{2}K_l(r-r_0)^2 \frac{1}{2}K_h(\theta-\phi)^2 - u^{\mathrm{T}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \end{pmatrix} q \end{aligned} \tag{22}$$

Let us consider the dynamics in the flight phase as below.

$$\begin{cases} \ddot{x} = 0 \\ \ddot{z} = -g \\ J_l\ddot{\theta} + J_b\ddot{\phi} = 0 \\ J_b\ddot{\phi} = K_h(\theta - \phi) + \tau_f \end{cases} \tag{23}$$

Here the variables x and z represent the horizontal and vertical positions of the center of mass and τ_f is the control torque.

4.2 Problem setting

This section sets the control problem to get the periodic gait based on (Hyon, 2005). Consider the behavior in a flight phase and let $\mathcal{J}$ denote the map from the initial state to the terminal state in this phase. This map can be regarded as a discrete map which connects two adjacent stance phases. Then the desired map of $\mathcal{J}$ is given by

$$\mathcal{J}_s := (r, \theta, \phi, p_r, p_\theta, p_\phi) \mapsto (r, -\theta, -\phi, -p_r, p_\theta, p_\phi) \tag{24}$$

We explain why such a map $\mathcal{J}_s$ in (24) is desired. Let us define the flow Φ_t in the stance phase with no inputs, i.e., $\rho = \tau = 0$ as $\Phi_t := (q(t^0), p(t^0)) \mapsto (q(t^0 + t), p(t^0 + t))$. From (22) and (24), the Hamiltonian $H(q, p, 0)$ is invariant with respect to $\mathcal{J}_s$. Therefore $\mathcal{J}_s$ and Φ_t satisfy

$$\mathcal{J}_s \circ \Phi_{T_s} = \mathrm{id} \tag{25}$$

If this equation holds, then a periodic gait is generated.

The leg angle θ is the most important state variable, because it has a direct effect to avoid falling. However, it is difficult to control the variable θ in the stance phase, since this robot has no foot. As in (Hyon, 2005), we apply no input in the stance phase, and try to control the variable θ to let $\mathcal{J} = \mathcal{J}_s$ hold in the flight phase.

In (Hyon and Emura, 2004), dead-beat control is used. It works well, but it requires the precise knowledge of the plant system. Here we try to use iterative learning control based on variational symmetry with a special cost function given in the following section. It will be shown to generate an optimal flow in the flight phase without the precise knowledge of the system.

4.3 Application of iterative learning control

Let us define the desired values of θ and $\dot{\theta}$ as follows (we let $t^0 = 0$ for simplicity in what follows.)

$$\theta^d := \theta\big|_{t=T_s+T_f} = -\theta\big|_{t=T_s} \tag{26}$$

$$\dot{\theta}^d := \dot{\theta}\big|_{t=T_s+T_f} = \dot{\theta}\big|_{t=T_s} \tag{27}$$

As for the model mentioned above, energy dissipation occurs at the touchdown. Let E_- and E_+ represent the energies just before the touchdown and just after the touchdown. Then the variation of the energy between them can be calculated as below from (Hyon and Emura, 2004)

$$E_+ - E_- = -\frac{mJ_l}{2(J_l + mr_0^2)}\mu_-^2 \tag{28}$$

where μ_- is defined as follows and is called the energy dissipation coefficient.

$$\mu_- := v_{x\_} cos\theta_- + v_{z\_} sin\theta_- + \frac{r_0}{J_l + mr_0^2} p_{\theta\_} \tag{29}$$

Here $v_{x\_}$ and $v_{z\_}$ represent the velocity of the center of mass. Suppose that the condition (30) holds at the touchdown. This implies that there is no energy transfer except for the control input.

$$\mu_- = 0 \tag{30}$$

If the total mechanical energy is completely preserved, it is expected that periodic gait trajectories are autonomously generated. In fact, (Hyon and Emura, 2004) implies that the condition (30) is satisfied if the control objects (26) and (27) are achieved, and the initial condition is appropriately chosen (the way how to choose is described in (Hyon and Emura, 2004).

Now we propose a novel cost function as

$$\Gamma(\theta, \dot{\theta}, u) := \frac{K_\theta}{2}\|\theta - (-\mathcal{R}(\theta))\|_{L_2}^2 + \frac{K_{\dot{\theta}}}{2}\|\dot{\theta} - \mathcal{R}(\dot{\theta})\|_{L_2}^2 + \frac{K_u}{2}\|u\|_{L_2}^2 \tag{31}$$

where K_θ, $K_{\dot{\theta}}$ and K_u represent appropriate positive constants. $\mathcal{R}$ is the time-reversal operator as defined in Section 2. The first term in the right hand side of (31) is expected to make Assumption 3 approximately hold. It is expected that we can generate an optimal trajectory such that it satisfies (26) and (27) minimizing the L_2 norm of the control input. Furthermore, there is no energy transfer except for the control input.

Let us recall the fact that gait trajectories are essentially periodic, however ILC can not generate periodic trajectories. Let us connect the stance flow Φ_t and that generated from (31). Take this connected trajectory as an one period of a periodic gait trajectory. Therefore if (30) holds, we can generate the optimal periodic trajectory.

Now, let us define input as $u = \tau_f$ and output as $y = \theta$. We can calculate the iteration law as in (7) and (20). (Some details are omitted due to limitations of space.)

$$u_{(2i-1)} = u_{(2i-2)} + 2\mathcal{R}\Big(K_\theta(\mathrm{id} + \mathcal{R})\mathrm{y}_{(2i-2)} + \mathrm{K}_{\dot{\theta}}(\mathrm{id} - \mathcal{R})\ddot{\mathrm{y}}_{(2i-2)}\Big) \tag{32}$$

$$u_{(2i)} = (\mathrm{id} - \mathrm{K}_{(2i-2)}\mathrm{K_u})\mathrm{u}_{(2i-2)} - \mathrm{K}_{(2i-2)}\mathcal{R}\Big(\mathrm{y}_{(2i-1)} - \mathrm{y}_{(2i-2)}\Big) \tag{33}$$

Here id represents the identity mapping.

5. SIMULATION

We apply the proposed iteration law of (32) and (33) to the hopping robot introduced in the previous section. We proceed 50 steps of the learning algorithm which means we execute 100 simulations.

Figure 3 shows that the cost function (31) almost decreases at each experiment. This implies that

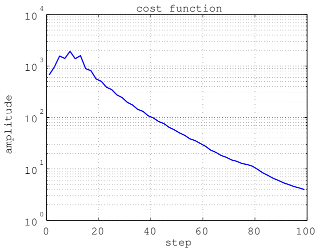

Fig. 3. Cost function

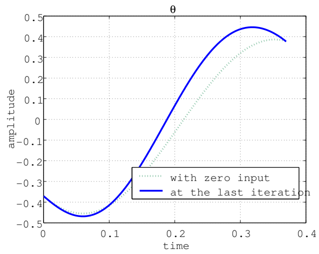

Fig. 4. Response of θ

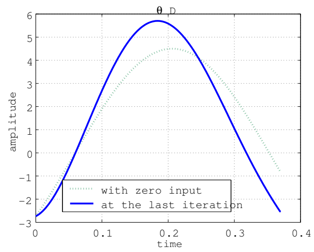

Fig. 5. Response of $\dot{\theta}$

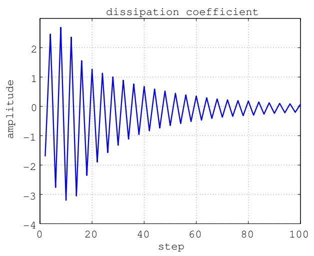

Fig. 6. Energy dissipation coefficient μ_-

the output trajectory converges to the optimal one smoothly. We choose the initial input $u_{(0)} \equiv 0$, so Assumption 3 does not hold at first. This is probably the reason why the cost function increases in the beginning of the learning procedure. Figure 4 and 5 show responses of θ and $\dot{\theta}$ at the last step in solid lines and the initial trajectories corresponding outputs with 0 input in dotted lines. These figures show that both θ and $\dot{\theta}$ converge to the trajectories satisfying (26) and (27). Furthermore from Figure 6, the energy dissipation coefficient (29) converges to zero as well.

6. CONCLUSION

In this paper, we have proposed an extension of the iterative learning control based on variational symmetry to use a pseudo adjoint of the time derivative operator. This allows one to execute iterative learning with optimal control type cost function including time derivatives of the output signal, which was not possible in the existing result. Application of this method to gait generation problem of a hopping robot derives an optimal gait trajectories without using precise knowledge of the plant. Finally, numerical simulations have demonstrated the effectiveness of the proposed method.

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