Periodic Gait Generation via Repetitive Optimal Control of Hamiltonian Systems

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Abstract: This paper proposes an optimal gait generation framework based on a property of Hamiltonian systems. A key technique is a unified method of learning control and parameter tuning. The proposed method allows one to simultaneously obtain an optimal feedforward input and an optimal tuning parameter for a plant system, which (at least locally) minimize a cost function. It is a repetitive control type optimal gait generation framework, since its iteration procedure is automatically executed and eventually an optimal periodic trajectory is generated.

1. INTRODUCTION

Recently, control of walking robots has become an active research area. As the technology for walking robots evolves, an optimization problem of gaits with respect to energy consumption becomes increasingly important. From this point of view, Passive Dynamic Walking (PDW) studied by McGeer (McGeer [1990]) attracts attention. Behavior analysis of PDW is studied by, e.g., (Osuka and Kirihara [2000], Sano et al. [2003], Garcia et al. [1998]). Energy-efficient walking control methods based on PDW have been proposed by many researchers, e.g. (Goswami et al. [1997], Spong [1999], Asano et al. [2004]). On the other hand, we consider that physical property and learning control are useful tools to tackle this challenging problem. Hamiltonian systems (Maschke and van der Schaft [1992]) have been introduced to represent physical systems and they explicitly possess good properties for control design. Iterative Learning Control (ILC) based on variational symmetry of Hamiltonian systems was proposed in (Fujimoto and Sugie [2003]) and it allows one to solve a class of optimal control problems by iteration of laboratory experiments. Thanks to the symmetric property, it does not require the precise knowledge of the plant model. Although this method works well for some control problems, there are mainly two difficulties to apply it to the optimal gait generation problem. The first one is that this method deals with a functional of the input and the output as a cost function but it can not take the time derivative of the output into account. This signal represents the generalized velocity of mechanical systems, which severely affects the walking motion. The other difficulty is that the conventional learning method can not take discontinuous state transitions into account. Such discontinuous changes involved in general walking motions also have to be considered. We have proposed an extended iterative learning control framework for the optimal gait generation in (Satoh et al. [2008a]) to solve these problems.

Since our previous method is classified as ILC framework, it requires a lot of laboratory experiments under the same initial condition as well as many conventional results, e.g. (Arimoto et al. [1984], Fujimoto and Sugie [2003]). This paper proposes a repetitive control (Hara et al. [1988]) type optimal gait generation framework based on repetitive optimal control of Hamiltonian systems. Repetitive optimal control proposed here unifies ILC (Fujimoto and Sugie [2003]) and Iterative Feedback Tuning (IFT) (Fujimoto and Koyama [2008]) based on variational symmetry of Hamiltonian systems, which is a modification of our previous concept in (Satoh et al. [2008b]). While ILC is to find an optimal feedforward input minimizing a given cost function, IFT is to find optimal parameters of a given feedback controller. The proposed framework is summarized as follows. Firstly, we add a constraint by adding a virtual potential energy to prevent the robot from falling. Secondly, we execute the learning procedure in (Satoh et al. [2008a]). The proposed technique restricts the motion of the robot to a symmetric trajectory by the virtual constraint. Due to this additional constraint, we do not need to repeat experiments under the same initial condition. Thirdly, by regarding the potential gain for the constraint as a tuning parameter, we execute IFT to mitigate the strength of the virtual constraint automatically according to the progress of learning control. Consequently, it is expected to generate an optimal gait without constraint eventually. Although this method requires one to apply ILC and IFT simultaneously, both methods influence each other and they regularly can not be used at a time. Our previous report in (Satoh et al. [2008b]) did not take the interference of ILC and IFT into account enough. To solve this problem, the proposed method introduces an extended system which again has variational symmetry. The extended system instead of the original plant system enables one to apply ILC and IFT simultaneously. Since

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the proposed learning procedure is automatically executed and eventually an optimal periodic trajectory is generated, it is classified as repetitive control framework.

2. PRELIMINARIES

This section briefly refers to preliminary backgrounds.

2.1 Hamiltonian systems and variational symmetry

Our plant is a Hamiltonian system $\Sigma^{x_{t^0}} : U \to Y : u \mapsto y$ with a controlled Hamiltonian $H(x, u, \rho)$ with parameters

$$\Sigma^{x_{t^0}} : \begin{cases} \dot{x} = (J - R)\dfrac{\partial H(x,u,\rho)}{\partial x}^\top, \ x(t^0) = x_{t^0} \\ y = -\dfrac{\partial H(x,u,\rho)}{\partial u}^\top \end{cases} . \quad (1)$$

Here, $x(t) \in \mathbb{R}^n$, $u(t), y(t) \in \mathbb{R}^m$ and $\rho \in \mathbb{R}^s$ describe the state, the input and the output and adjustable parameters, respectively. In this paper, we consider U, Y as $L_2^m[t^0, t^1]$. The structure matrix $J \in \mathbb{R}^{n\times n}$ and the dissipation matrix $R \in \mathbb{R}^{n\times n}$ are skew-symmetric and symmetric positive semi-definite, respectively. The variational system $\delta\Sigma^{x_{t^0}}$ of the system $\Sigma^{x_{t^0}}$ represents the Fréchet derivative of $\Sigma^{x_{t^0}}$. According to (Fujimoto and Sugie [2003]), under certain conditions the adjoint of the variational system $(\delta\Sigma^{x_{t^0}})^*$ has the following relationship with the variational system:

$$(\delta\Sigma^{x_{t^0}}(u))^*(v) = \mathcal{R}(\delta\Sigma^{\phi_{t^0}}(w))\mathcal{R}(v)$$
$$\approx \frac{1}{\epsilon}\mathcal{R} \circ (\Sigma^{\phi_{t^0}}(w + \epsilon\mathcal{R}(v)) - \Sigma^{\phi_{t^0}}(w)), \quad (2)$$

where ϕ_{t^0} and w denote appropriate initial condition and input, respectively, ϵ represents sufficiently small positive constant and $\mathcal{R}$ is a time-reversal operator defined by $\mathcal{R}(u)(t - t^0) = u(t^1 - t)$ for $\forall t \in [t^0, t^1]$. This property is called **variational symmetry** of Hamiltonian systems. Approximation (2) implies that one can calculate the input-output mapping of the adjoint by only using the input-output data of the original system.

2.2 Iterative learning control (ILC) and iterative feedback tuning (IFT) based on variational symmetry

This subsection refers to ILC in (Fujimoto and Sugie [2003]) and IFT in (Fujimoto and Koyama [2008]). They have common feature that they take advantage of variational symmetry of Hamiltonian systems mentioned in subsection 2.1. The objective of ILC is to find an optimal feedforward input which minimizes a given cost function, while that of IFT is to find optimal parameters of a given feedback controller. Firstly, let us mention ILC in more detail. Consider the system $\Sigma^{x_{t^0}}$ in (1) and a cost function $\hat{\Gamma}(u, y) : U \times Y \to \mathbb{R}$. This technique is based on the steepest descent method. The gradient of the cost function $\hat{\Gamma}$ with respect to the control input u is calculated as

$$\delta\hat{\Gamma}(u, y)(\delta u, \delta y) = \langle \nabla_u\hat{\Gamma}(u, y), \delta u\rangle_U + \langle \nabla_y\hat{\Gamma}(u, y), \delta y\rangle_Y$$
$$= \langle \nabla_u\hat{\Gamma} + (\delta\Sigma^{x_{t^0}}(u))^*(\nabla_y\hat{\Gamma}), \delta u\rangle_U =: \langle \nabla_u\Gamma(u), \delta u\rangle_U, \quad (3)$$

where $\Gamma(u) = \hat{\Gamma}(u, \Sigma^{x_{t^0}}(u))$. It follows from well-known Riesz's representation theorem that there exists functions $\nabla_u\hat{\Gamma}(u, y)$ and $\nabla_y\hat{\Gamma}(u, y)$ as above. The steepest descent method implies that we should update the input u as

$$u_{(i+1)} = u_{(i)} - K_{(i)}\nabla_u\Gamma(u_{(i)}), \ i = 0, 1, 2, \cdots, \quad (4)$$

where $K_{(\cdot)}$'s are appropriate positive matrices and i denotes the i-th iteration in laboratory experiment. In calculating the gradient of the cost function $\nabla_u\Gamma$, the precise knowledge of the plant system is generally required to obtain $(\delta\Sigma^{x_{t^0}}(u))^*$. However, ILC in (Fujimoto and Sugie [2003]) takes advantage of variational symmetry and then approximation (2) allows one to realize the update law (4) without precise information of the plant system.

Secondly, IFT proposed in (Fujimoto and Koyama [2008]) is mentioned. In this method, tuning parameters of a given feedback controller are considered to be virtual inputs for a Hamiltonian system to utilize variational symmetry. The algorithm is similar to that of ILC. We define the zeroth-order hold operator $\mathfrak{h}$ which maps the parameter $\rho \in \mathbb{R}^s$ to a virtual input $u_\rho \in L_2^s[t^0, t^1]$ as $u_\rho(t) := (\mathfrak{h}(\rho))(t) \equiv \rho, \ \forall t \in [t^0, t^1]$. Then the corresponding output which induces variational symmetry (2) is given by $y_\rho := -\frac{\partial H(x,u,\rho)}{\partial \rho}^\top$. Utilizing u_ρ and y_ρ, one can update parameters in the similar manner (4) as in the case of ILC.

3. DESCRIPTION OF THE PLANT

Let us consider a full-actuated planar compass-like biped robot with a torso depicted in Fig. 1. 1-period of walking

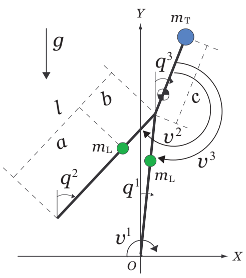

Fig. 1. Model of the compass gait biped with a torso

describes the period between the take-off of one foot from the ground and its subsequent landing. Table 1 shows physical parameters and variables. In this paper, we define

Table 1. Parameters and variables

Notation	Meaning
$m_T = 5.0$kg	torso mass
$m_L = 1.2$kg	leg mass
$a = 0.2$m	length from m_L to the ground
$b = 0.2$m	length from the hip to m_L
$l = a + b$	total leg length
$c = 0.12$m	length from the hip to m_T
$g = 9.81$m/s^2	gravity acceleration
q^1	stance leg angle w.r.t vertical
q^2	swing leg angle w.r.t vertical
q^3	torso angle w.r.t vertical
v^1	ankle torque
v^2	torque applied between torso and swing leg
v^3	torque applied between torso and stance leg

the input as $u = (u^1, u^2, u^3)^\top := (v^1 - v^3, -v^2, v^2 + v^3)^\top$ in order to simplify the input-output relation in the

Hamiltonian form (5) mentioned later. Furthermore, We assume the following on this robot.

Assumption 1. The foot of the swing leg does not bounce back nor slip on the ground at the collision.

Assumption 2. Transfer of support between the stance and the swing legs is instantaneous.

Assumption 3. The foot-scuffing during the single support phase can be ignored.

A typical mechanical system can be described by a Hamiltonian system (1) with the state $x=(q^\top,p^\top)^\top\in\mathbb{R}^{2m}$ as

$$\begin{pmatrix}\dot q\\ \dot p\end{pmatrix}=\left(\begin{pmatrix}O_{mm} & I_m\\ -I_m & O_{mm}\end{pmatrix}-\begin{pmatrix}O_{mm} & O_{mm}\\ O_{mm} & R_D\end{pmatrix}\right)\begin{pmatrix}\dfrac{\partial H(x,u)}{\partial q}^\top\\ \dfrac{\partial H(x,u)}{\partial p}^\top\end{pmatrix}$$

$$y=-\frac{\partial H(x,u)}{\partial u}^\top=q \tag{5}$$

with the Hamiltonian $H(x,u)=\frac{1}{2}p^\top M(q)^{-1}p+V(q)-u^\top q$. Here q represents the configuration coordinate and a positive definite matrix $M(q)\in\mathbb{R}^{m\times m}$ denotes the inertia matrix. The generalized momentum $p\in\mathbb{R}^m$ is given by $p:=M(q)\dot q$. A positive semi-definite matrix $R_D\in\mathbb{R}^{m\times m}$ denotes the friction coefficients, and a scalar function $V(q)\in\mathbb{R}$ denotes the potential energy of the system. The dynamics of the robot depicted in Fig.1 is described as a typical mechanical system in (5) with $m=3$, the friction coefficients $R_D=O_{33}$. For the details of $M(q)$ and $V(q)$, see (Satoh [2010]). At the end of a walking period, a collision between a leg and the ground causes a discontinuous change in angular velocities. Assumptions 1 and 2 imply that there exists no double support phase. Since the support and swing legs change each other instantly, we have

$$q_+=\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix}q_-=:Cq_-, \tag{6}$$

where q_- and q_+ denote the angles just before and after the collision, respectively. Although a transition mapping can be derived by the conservation law of the angular momentum, see (Grizzle et al. [2001]) for the detail.

Before the ILC method mentioned in Subsection 2.2 is applied, feedback controllers are typically employed to the control system in order to render the system asymptotically stable. In (Fujimoto and Sugie [2001]), a generalized canonical transformation, which is a pair of feedback and coordinate transformations preserving the Hamiltonian structure of the form (1), is proposed. It is known that in the case of a typical mechanical system in (5), a simple PD feedback preserves the Hamiltonian structure (Fujimoto and Sugie [2003]). Let us consider a PD controller

$$u=-K_Pq-K_D\dot q+\bar u, \tag{7}$$

where $\bar u$ is a new input for ILC and $K_P,K_D\in\mathbb{R}^{3\times3}$ are symmetric positive definite matrices. In what follows, we consider the feedback system with a new Hamiltonian $H_c(x,\bar u,\rho)$ of the form (5) by a PD controller (7) with sufficiently large gains K_P and K_D.

4. MAIN RESULTS

This section proposes an optimal periodic gait generation framework via repetitive optimal control. Subsection 4.1 proposes a unified method of ILC and IFT of Hamiltonian systems, which plays an important role in the proposed framework. In order to take interference of ILC and IFT into account, this method introduces an extended system which again has variational symmetry. The extended system instead of the original plant system enables one to apply ILC and IFT simultaneously. In subsection 4.2, we equip a constraint by adding a virtual potential energy. Then, the concept of the proposed framework is outlined. In subsection 4.3, we define a cost function and exhibit a proposed algorithm.

4.1 Learning optimal control unifying ILC and IFT

Let us define the extended input u_e by $u_e:=(\bar u^\top,u_\rho^\top)^\top\in U_e=U\times U_\rho$, the extended output y_e by $y_e:=(y^\top,y_\rho^\top)^\top\in Y_e=Y\times Y_\rho$, where $U_\rho,Y_\rho=L_2^s[t^0,t^1]$ and Hamiltonian H_e by $H_e(x,u_e):=H_c(x,\bar u,u_\rho)$. Then we have the following extended system $y_e=\Sigma_e^{x_{t^0}}(u_e)$:

$$\begin{cases}\dot x\ =(J-R)\dfrac{\partial H_e(x,u_e)}{\partial x}^\top,\ x(t^0)=x_{t^0}\\ y_e\ =-\dfrac{\partial H_e(x,u_e)}{\partial u_e}^\top\end{cases}. \tag{8}$$

Since the extended system (8) has the form (1), it can be easily proven that the system has variational symmetry with certain conditions. Now we consider a cost function $\hat\Gamma_e(u_e,y_e):U_e\times Y_e\to\mathbb{R}$. From the definition of u_e, we have

$$\delta u_e=\begin{pmatrix}\delta\bar u\\ \delta u_\rho\end{pmatrix}=\begin{pmatrix}\delta\bar u\\ \delta\mathfrak{h}(\rho)\,\mathrm{d}\rho\end{pmatrix}=\begin{pmatrix}\delta\bar u\\ \mathfrak{h}(\mathrm{d}\rho)\end{pmatrix}. \tag{9}$$

The last equality follows from the linearity of the Fréchet derivative and the operator $\mathfrak{h}$. From Eq. (9), The Fréchet derivative of the cost function can be calculated as

$$\begin{aligned}&\delta\hat\Gamma_e(u_e,y_e)(\delta u_e,\delta y_e)\\ &=\left\langle\begin{pmatrix}\mathrm{id}&0\\0&\mathfrak{h}^*\end{pmatrix}\left(\nabla_{u_e}\hat\Gamma_e+(\delta\Sigma_e^{x_{t^0}}(u_e))^*(\nabla_{y_e}\hat\Gamma_e)\right),\begin{pmatrix}\delta\bar u\\ \mathrm{d}\rho\end{pmatrix}\right\rangle_{U\times\mathbb{R}^s}\\ &=\left\langle\begin{pmatrix}\nabla_{\bar u}\hat\Gamma_e\\ \mathfrak{h}^*(\nabla_{u_\rho}\hat\Gamma_e)\end{pmatrix}+\begin{pmatrix}\mathrm{id}&0\\0&\mathfrak{h}^*\end{pmatrix}\mathcal{R}(\delta\Sigma_e^{\psi_{t^0}}(w_e))\mathcal{R}(\nabla_{y_e}\hat\Gamma_e)\right.\\ &\qquad\left.,\begin{pmatrix}\delta\bar u\\ \mathrm{d}\rho\end{pmatrix}\right\rangle_{U\times\mathbb{R}^s}\\ &\approx\left\langle\begin{pmatrix}\nabla_{\bar u}\hat\Gamma_e\\ \mathfrak{h}^*(\nabla_{u_\rho}\hat\Gamma_e)\end{pmatrix}+\begin{pmatrix}\mathcal{R}&0\\0&\mathfrak{h}^*\end{pmatrix}\times\right.\\ &\left.\left(\frac{\Sigma_e^{\psi_{t^0}}(w_e+\epsilon_e\mathcal{R}(\nabla_{y_e}\hat\Gamma_e))-\Sigma_e^{\psi_{t^0}}(w_e)}{\epsilon_e}\right),\begin{pmatrix}\delta\bar u\\ \mathrm{d}\rho\end{pmatrix}\right\rangle_{U\times\mathbb{R}^s},\end{aligned} \tag{10}$$

where id represents the identity mapping. In the last approximation, the relation $\mathfrak{h}^*\mathcal{R}=\mathfrak{h}^*\mathcal{R}^*=(\mathcal{R}\mathfrak{h})^*=\mathfrak{h}^*$ is utilized (note that it follows from the definition of $\mathcal{R}$ that $\mathcal{R}^*=\mathcal{R}$). ψ_{t^0} and $w_e:=(w^\top,\mathfrak{h}(\rho)^\top)^\top$ in (10) should be chosen so that conditions for variational symmetry hold.

Remark 1. State trajectories under which, for $\forall t\in[t^0,t^1]$, q and $\dot q$ satisfy

$$q(t)=q(t^1-t+t^0),\quad \dot q(t)=-\dot q(t^1-t+t^0) \tag{11}$$

represent time-symmetric motions with respect to the middle point of time $t=(t^0+t^1)/2$. We call trajectories satisfying the condition (11) symmetric trajectories.

Suppose a state trajectory ϕ corresponding an input v is symmetric one. Then, in a typical mechanical system (5), conditions for variational symmetry in (2) are satisfies with sufficiently large PD gains under $\psi = \phi$ and $w = v$.

Suppose that the learning procedure is executed around a symmetric trajectory and a trajectory in each experiment approximately satisfies the condition (11), then the proposed algorithm combining ILC and IFT is given by

$$\begin{cases} x_{t^0(2i+1)} = x_{t^0(2i)} \\ \bar{u}_{(2i+1)} = \bar{u}_{(2i)} + \epsilon_{e(i)} \mathcal{R}(\nabla_y \hat{\Gamma}_{e(2i)}) \\ u_{\rho(2i+1)} = \mathfrak{h}(\rho_{(2i)}) + \epsilon_{e(i)} \mathcal{R}(\nabla_{y_\rho} \hat{\Gamma}_{e(2i)}) \end{cases} \tag{12}$$

$$\begin{cases} x_{t^0(2i+2)} = x_{t^0(2i)} \\ \bar{u}_{(2i+2)} = \bar{u}_{(2i)} - K_{(i)} \left(\nabla_{\bar{u}} \hat{\Gamma}_{e(2i)} + \dfrac{1}{\epsilon_{e(i)}} \mathcal{R}(y_{(2i+1)} - y_{(2i)}) \right) \\ \rho_{(2i+2)} = \rho_{(2i)} - K_{\rho(i)} \left(\displaystyle\int_{t^0}^{t^1} \nabla_{u_\rho} \hat{\Gamma}_{e(2i)} \right. \\ \qquad \left. + \dfrac{1}{\epsilon_{e(i)}} \mathcal{R}(y_{\rho(2i+1)} - y_{\rho(2i)})\, \mathrm{d}t \right), \end{cases}$$

provided that the initial control input $\bar{u}_{(0)} \equiv 0$ or an appropriate initial input, the initial parameter $\rho_{(0)}$ and the initial condition $x_{t^0(0)}$ are appropriately chosen, respectively. Here $\epsilon_{e(\cdot)}$'s denote sufficiently small positive constants and appropriate positive definite matrices $K_{(\cdot)}$'s and $K_{\rho(\cdot)}$'s represent gains, respectively.

The proposed algorithm implies that the learning procedure needs two experiments to execute a single update in (4). Firstly, in the $(2i+1)$-th iteration, we calculate the output $\Sigma^{\psi_{t^0}}(w_e + \epsilon_e \nabla_{y_e} \hat{\Gamma}_e)$ in Eq. (10) (note that in this case ψ_{t^0} corresponds to $x_{t^0(2i)}$). Then the input and output signals of $\delta\Sigma_e^{x_{t^0}}(u_e)^*(\nabla_{y_e}\hat{\Gamma}_e)$ can be calculated from the last approximation in Eq. (10). With this information, the gradient of the cost function with respect to the input $\nabla\Gamma_e(u_e)$ with $\Gamma_e(u_e) := \hat{\Gamma}_e(u_e, \Sigma_e(u_e))$(see also Eq. (3)) is obtained. Finally, the input for the $(2i+2)$-th iteration is given by Eq. (4) with these signals. The proposed method allows one to simultaneously obtain an optimal feedforward input and an optimal tuning parameter.

Remark 2. In the proposed framework, we restrict the motion of the robot to a symmetric trajectory by utilizing a virtual constraint mentioned in subsection 4.2 so that the algorithm (12) is valid. However, we have provided an algorithm for more general cases in (Satoh [2010]).

4.2 Gait generation framework via repetitive optimal control

In the literatures (Grizzle et al. [2001], Hyon and Emura [2005]), walking control methods using virtual constraints based on the output zeroing control are proposed. In (Hyon and Emura [2005]), particularly, they can achieve stable symmetric walking gaits, by which they set the output function $y := q^1 + q^2$ to zero by the output zeroing control and keep the leg angles bounded by a leg exchange scheme. As a consequence, they guarantee that the robot does not fall and obtain symmetric gaits satisfying $q^1 + q^2 = 0$.

On the other hand, we have proposed a similar idea of the virtual constraint to prevent the robot from falling, but we do not use the output zeroing control in (Satoh et al. [2008b]). There are two reasons: one is that the output zeroing control requires the precise knowledge of the plant system and the other is that such constraints consume a lot of control energy. Instead, we have equipped a virtual potential energy $P_c := \frac{k_c}{2}(q^1 + q^2)^2$ to produce a similar effect to (Hyon and Emura [2005]). Here, the gain parameter k_c represents the constraint strength. We let k_c sufficiently large at the beginning of learning steps so that the trajectory of the robot is restricted to a symmetric one, i.e. $q^1 + q^2 = 0$ holds. Due to (Hyon and Emura [2005]), it is expected that the robot does not fall. As advantages of the method, firstly, it does not require the model parameters of the plant system, since the potential energy P_c can be generated by a simple feedback controller

$$u = -K_P q - K_D \dot{q} + \bar{u} - k_c A_c q, \quad A_c := \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{13}$$

The feedback system is depicted in Fig. 2, where q^r and $\dot{q}^r$ represent reference signals for PD feedback, respectively. The controller in (13) corresponds to the case where $q^r \equiv \dot{q}^r \equiv 0$. Secondly, after adding the potential energy, the plant system preserves the Hamiltonian structure and the constraint parameter k_c is explicitly contained in a new Hamiltonian. The controller (13) converts the dynamics of the robot into another Hamiltonian system of the form (5) with a new Hamiltonian $\bar{H}(x, \bar{u}, k_c)$, a new structure matrix $\bar{J}$ and a new dissipation matrix $\bar{R}$ as

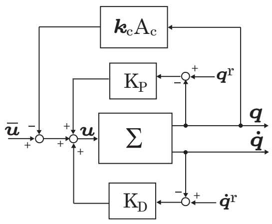

Fig. 2. Feedback system

$$\bar{H} = \frac{1}{2} p^\top M(q)^{-1} p + V(q) + \frac{1}{2} q^\top (K_P + k_c A_c) q - \bar{u}^\top q,$$
$$\bar{J} = \begin{pmatrix} O_{33} & I_3 \\ -I_3 & O_{33} \end{pmatrix}, \qquad \bar{R} = \begin{pmatrix} O_{33} & O_{33} \\ O_{33} & K_D \end{pmatrix}. \tag{14}$$

By regarding k_c as a tuning parameter, we execute the proposed learning technique unifying ILC and IFT in subsection 4.1, in order to adjust the constraint strength by IFT, and simultaneously generate a walking trajectory by ILC. The concept of the proposed gait generation framework is summarized as follows.

Step 1 : Add a virtual potential energy to restrict the motion of the robot to a symmetric trajectory. Then, let constraint parameter k_c sufficiently large to expect that the robot does not fall.

Step 2 : By utilizing optimal learning control scheme proposed in subsection 4.1, ILC generates an optimal walking gait and simultaneously, IFT mitigates the constraint parameter automatically according to the progress of learning control.

Step 3 : Repeat Step 2 every one step cycle.

It is expected that an optimal periodic gait is generated without constraint eventually. The feature of the framework is that the robot improves its walk keeping on walking, because it does not fall due to Step1. From this aspect, our method is classified as repetitive control framework rather than ILC one, so we call it gait generation framework via repetitive optimal control. It also differs from the conventional methods using virtual constraints in that it automatically optimizes the strength of the constraints.

4.3 Derivation of the iteration law

Let us consider the following cost function $\hat{\Gamma}(y, \dot{y}, \bar{u}, y_\rho, u_\rho)$:

$$\frac{1}{2}\int_{t^0}^{t^1} (y_e(\tau) - C_e\mathcal{R}(y_e)(\tau))^\top \Lambda_{y_e}(\tau)(y_e(\tau) - C_e\mathcal{R}(y_e)(\tau))\,\mathrm{d}\tau$$
$$+\frac{1}{2}\int_{t^0}^{t^1} F_v(\dot{y}_e(\tau) - v_{e,ref})^\top \Lambda_{\dot{y}_e}(\tau) F_v(\dot{y}_e(\tau) - v_{e,ref})\,\mathrm{d}\tau$$
$$+\frac{1}{2}\int_{t^0}^{t^1} u_e(\tau)^\top \Lambda_{u_e} u_e(\tau)\,\mathrm{d}\tau \;=: \hat{\Gamma}_e(y_e, \dot{y}_e, u_e), \qquad (15)$$

where $C_e := \mathrm{diag}\{C, 0\}$, $\Lambda_{y_e}(t) := \mathrm{diag}\{\nu_1(t)\Lambda_y, \gamma_{y_\rho}\}$, $\Lambda_{\dot{y}_e}(t) := \mathrm{diag}\{\nu_2(t)\Lambda_{\dot{y}}, 0\}$ and $\Lambda_{u_e} := \mathrm{diag}\{\Lambda_{\bar{u}}, \gamma_{u_\rho}\} \in \mathbb{R}^{4\times 4}$ and $v_{e,ref} := \mathrm{diag}\{v_{ref}, 0\} \in \mathbb{R}^4$. Appropriate positive definite matrices $\Lambda_y, \Lambda_{\dot{y}}, \Lambda_{\bar{u}} \in \mathbb{R}^{3\times 3}$, and positive constants γ_{y_ρ} and γ_{u_ρ} represent weight matrices and coefficients, respectively. The first term in (15) is a necessary condition for a periodic trajectory such that $q^1(t^0) \equiv q^2(t^1)$ and $q^2(t^0) \equiv q^1(t^1)$. Although another necessary condition with respect to $\dot{q}$ can be utilized as in (Satoh et al. [2008a]), where initial angular velocities are equivalent to those just after touch down, it is not equipped here for simplicity. In the second term, $v_{ref} \in \mathbb{R}^3$ represents a constant reference angular velocity, $\nu_1(t), \nu_2(t) \in \mathbb{R}$ denote filter functions defined respectively by

$$\nu_1(t) := \begin{cases} \frac{1}{2}\left(1 - \cos\left(\frac{t^0 + \Delta t - t}{\Delta t}\pi\right)\right) & (t^0 \le t \le t^0 + \Delta t) \\ 0 & (t^0 + \Delta t < t \le t^1) \end{cases}$$

$$\nu_2(t) := \begin{cases} 0 & (t^0 \le t < \frac{t^1 - t^0}{2} - \Delta\bar{t}) \\ \frac{1}{2}\left(1 - \cos\left(\frac{-\frac{t^1 - t^0}{2} + \Delta\bar{t} + t}{\Delta\bar{t}}\pi\right)\right) & (\frac{t^1 - t^0}{2} - \Delta\bar{t} \le t < \frac{t^1 - t^0}{2}) \\ \frac{1}{2}\left(1 - \cos\left(\frac{\frac{t^1 - t^0}{2} + \Delta\bar{t} - t}{\Delta\bar{t}}\pi\right)\right) & (\frac{t^1 - t^0}{2} \le t < \frac{t^1 - t^0}{2} + \Delta\bar{t}) \\ 0 & (\frac{t^1 - t^0}{2} + \Delta\bar{t} \le t \le t^1) \end{cases},$$

where design parameters Δt and $\Delta\bar{t}$ denote positive constants. Figure 3 illustrates $\nu_1(t)$ and $\nu_2(t)$. For any

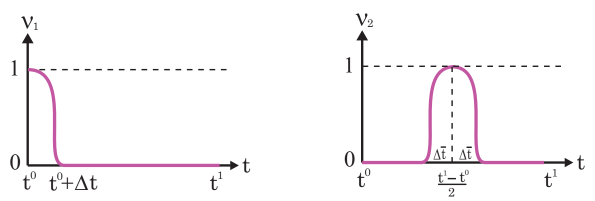

Fig. 3. Filter functions ν_1 and ν_2

$\zeta \in \mathbb{R}^4$, a penalty function $F_v : \mathbb{R}^4 \to \mathbb{R}^4$ is defined as

$$[F_v(\zeta)]^i = \begin{cases} k_{F_v}(\zeta^i)^2 & \text{if } \zeta^i < 0 \\ 0 & \text{otherwise} \end{cases}, \quad (i = 1, 2, 3, 4), \quad (16)$$

where an appropriate positive constant k_{F_v} represents the strength of penalty. The second term in (15) encourages the robot to achieve an appropriate velocity in the middle of walking. As a consequence, it is aimed at specifying the walking direction (forward or backward) and a rough walking speed, and preventing the robot from stopping during the learning. The last term is to minimize the control input and the strength of the virtual constraint, respectively.

Due to the virtual constraint equipped in Subsection 4.2, it is supposed that the learning procedure is executed around a symmetric trajectory. We summarize the proposed procedure. For the details of derivation, see (Satoh [2010]).

Step 0 : Set appropriately $\Lambda_y, \Lambda_{\dot{y}}$, $\Lambda_{\bar{u}}$, γ_{y_ρ}, γ_{u_ρ}, Δt, $\Delta\bar{t}$ and k_{F_v}. Set the initial input $\bar{u}_{(0)}$ appropriately (or set $\bar{u}_{(0)} \equiv 0$) and a constant reference angular velocity v_{ref}. Let the constraint parameter $k_{c(0)}$ sufficiently large and let the robot start walking under an appropriate initial condition x_{t^0}. Set $i = 0$. Then go to Step 1.

Step $2i + 1$: During the $(2i+1)$-th walking cycle, one utilizes the following controller:

$$u = -K_P q - K_D \dot{q} - u_{\rho(2i+1)} A_c q + \bar{u}_{(2i+1)},\; A_c := \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Here the feedback gain for the virtual constraint $u_{\rho(2i+1)}$ and the feedforward control input $\bar{u}_{(2i+1)}$ are given by

$$\begin{cases} \bar{u}_{(2i+1)} = \bar{u}_{(2i)} + \epsilon_{e(i)}\mathcal{R}(\nabla_y \hat{\Gamma}_{e(2i)}) \\ u_{\rho(2i+1)} = u_{\rho(2i)} + \epsilon_{e(i)}\mathcal{R}(\nabla_{y_\rho}\hat{\Gamma}_{e(2i)}) \end{cases}$$

with a sufficiently small positive constant $\epsilon_{e(i)}$ and

$$\nabla_y \hat{\Gamma}_{e(2i)} = (\mathrm{id} - \mathcal{R}C)\nu_1\Lambda_y(\mathrm{id} - C\mathcal{R})(y_{(2i)})$$
$$- \frac{\mathrm{d}}{\mathrm{d}t}((\delta F_v(\dot{y}_{(2i)} - v_{ref}))^* \nu_2 \Lambda_{\dot{y}} F_v(\dot{y}_{(2i)} - v_{ref})),$$
$$\nabla_{y_\rho}\hat{\Gamma}_{e(2i)} = \gamma_{y_\rho} y_{\rho(2i)}.$$

Step $2i + 2$: During the $(2i+2)$-th walking cycle, one utilizes the following controller:

$$u = -K_P q - K_D \dot{q} - k_{c(2i+2)} A_c q + \bar{u}_{(2i+2)}.$$

Here the feedback gain $k_{c(2i+2)}$ which represents the strength of the virtual constraint and the feedforward control input $\bar{u}_{(2i+2)}$ are given by

$$\begin{cases} \bar{u}_{(2i+2)} = \bar{u}_{(2i)} - K_{(i)}\left(\Lambda_{\bar{u}}\bar{u}_{(2i)} + \frac{1}{\epsilon_{e(i)}}\mathcal{R}(y_{(2i+1)} - y_{(2i)})\right) \\ k_{c(2i+2)} = k_{c(2i)} - K_{\rho(i)}\Big(\gamma_{u_\rho} k_{c(2i)}(t^1 - t^0) \\ \qquad + \frac{1}{\epsilon_{e(i)}}\int_{t^0}^{t^1} \mathcal{R}(y_{\rho(2i+1)} - y_{\rho(2i)})\,\mathrm{d}t\Big) \end{cases},$$

where appropriate positive definite matrix $K_{(i)}$ and positive constant $K_{\rho(i)}$ represent learning and tuning gains, respectively. Set $i = i+1$. Then, go to Step $2i+1$.

5. NUMERICAL EXAMPLE

We apply the proposed algorithm to the compass gait biped with a torso depicted in Fig. 1. The following PD feedback gains are utilized: $K_P = \mathrm{diag}(4, 4, 6)$ and $K_D = \mathrm{diag}(2, 2, 4)$. In this simulation, we assign a reference velocity only to $\dot{q}^1$ as $\tilde{v}_{ref} = (0.5, 0, 0)^\top$, since $\dot{q}^1$ mainly affects the walking velocity. We utilize the following design parameters: $\Lambda_y = \mathrm{diag}(20, 20, 20)$, $\Lambda_{\dot{y}} = \mathrm{diag}(10, 0, 0)$, $\gamma_{y_\rho} = 1 \times 10^{-2}$, $\Lambda_{\bar{u}} = \mathrm{diag}(1 \times 10^{-4}, 5 \times 10^{-5}, 5 \times 10^{-5})$, $\gamma_{u_\rho} = 1 \times 10^{-2}$, $\Delta t = 5.0 \times 10^{-3}$, $\Delta\bar{t} = 0.1$,

$k_{F_v} = 0.25$, $K_{(\cdot)} = \text{diag}(3,3,3)$, $K_{\rho(\cdot)} = 1$ and $\epsilon_{e(\cdot)} = 1$. We proceed 500 steps of the learning procedure with $k_{c(0)} = 30$, $(q_{t^0}^\top, \dot{q}_{t^0}^\top) = (-0.18, 0.20, 0, 1.1, 0.5, 0)$ and $\bar{u}_{(0)}(t) \equiv (0.5, -1.5)^\top$. Figure 4 shows the history of the cost function (15) along the walking steps. Since it monotonically decreases and then converges to a constant value, it implies that at least a local minimum of the cost function is achieved smoothly. Figure 5 shows the history of the constraint parameter k_c along the walking steps. It implies that the strength of constraint is adjusted. Although k_c does not converge to zero, it plays a role of a stabilizing feedback controller. Figures 6 and 7 represent the animations of the robot in the first and the last 5 steps, respectively. They show that at the beginning the robot walks awkwardly and then it improves its walk. Figure 8 shows that the phase portrait of q-$\dot{q}$ forms closed orbits. It implies that a periodic trajectory is generated.

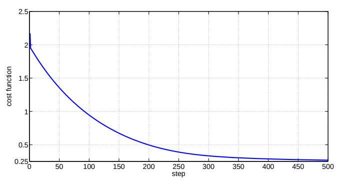

Fig. 4. Cost function

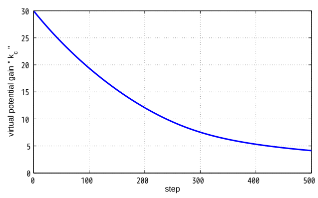

Fig. 5. Parameter k_c

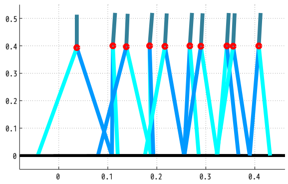

Fig. 6. Stick diagrams in the first 5 steps

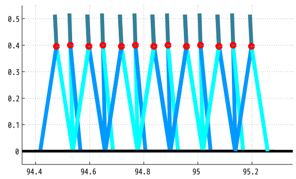

Fig. 7. Stick diagrams in the last 5 steps

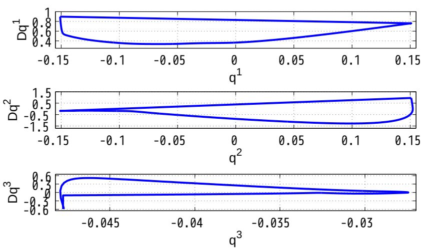

Fig. 8. Phase portrait of q-$\dot{q}$

6. CONCLUSION

This paper has proposed a repetitive control type optimal gait generation framework. The proposed unified method of ILC and IFT of Hamiltonian systems plays a key role and it allows one to simultaneously obtain optimal feedforward input and tuning parameter, which minimize a cost function.

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