Gait Generation for a Hopping Robot via Iterative Learning Control Based on Variational Symmetry

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Abstract: This paper proposes a novel framework to generate optimal gait trajectories for a one-legged hopping robot via iterative learning control. This method generates gait trajectories which are solutions of a class of optimal control problems without using precise knowledge of the plant model. It is expected to produce natural gait movements such as that of a passive walker. Some numerical examples demonstrate the effectiveness of the proposed method.

1 Introduction

Passive dynamic walking originally studied by McGeer [1] inspires many researchers to work on a gait generation problem for walking robots. They try to design more natural and less energy consuming gait trajectories than those produced by conventional walking control such as ZMP based control. Behavior analysis of passive walkers were investigated, e.g, in [2, 3]. There are some results on gait generation based on passive dynamic walking [4, 5, 6] by designing appropriate feedback control systems such that the closed loop systems behave like passive walkers. In particular, a hopping robot modelled in [7] has a hopping gait for which the input signal coincides with zero, that is, this robot can be regarded as a passive walker walking on a horizontal plane. Furthermore, an adaptive control system for this robot to achieve a walking gait with zero input was proposed in the authors' former result [8].

The objective of this paper is to generate optimal walking gait trajectories for a hopping robot via iterative learning control without using precise knowledge of the plant model. To this end, we formulate an optimal control type cost function and try to find a control input minimizing it by iterative learning technique based on *variational symmetry* of Hamiltonian control systems [9], which can solve a class of optimal control problems by iteration of experiments. For this purpose, two novel techniques with respect to iterative learning control are proposed: One is a technique to take the time derivatives of the output signal into account in the iterative learning control by employing a pseudo adjoint of the time derivative operator. The other is a cost function to achieve time symmetric gait trajectories to guarantee stable walking without a fall. Furthermore, the proposed learning scheme is applied to the hopping robot in [7] and the corresponding numerical simulations demonstrate its advantage.

2 Iterative learning control based on variational symmetry

This section refers to the iterative learning control (ILC) method based on variational symmetry in [9] briefly.

2.1 Variational symmetry of Hamiltonian systems

Consider a Hamiltonian system with dissipation and a controlled Hamiltonian $H(x, u, t)$ described by

$$\Sigma : \begin{cases} \dot{x} = (J - R)\dfrac{\partial H(x, u, t)}{\partial x}^{\mathrm{T}}, \quad x(t^0) = x^0 \\ y = -\dfrac{\partial H(x, u, t)}{\partial u}^{\mathrm{T}} \end{cases} . \tag{1}$$

Here $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$ and $y(t) \in \mathbb{R}^r$ describe the state, the input and the output, respectively. The structure matrix $J \in \mathbb{R}^{n \times n}$ and the dissipation matrix $R \in \mathbb{R}^{n \times n}$ are skew-symmetric and symmetric positive semi-definite, respectively. The matrix R represents dissipative elements. In this paper the Hamiltonian system in (1) is written as $y = \Sigma(u)$. For this system, the following theorem holds. This property is called *variational symmetry* of Hamiltonian control systems.

Theorem 1. *[9] Consider the Hamiltonian system in (1). Suppose that J and R are constant and that there exists a nonsingular matrix $T \in \mathbb{R}^{n \times n}$ satisfying*

$$J = -TJ\,T^{-1}$$
$$R = TR\,T^{-1}$$
$$\frac{\partial^2 H(x,u,t)}{\partial(x,u)^2} = \begin{pmatrix} T & 0 \\ 0 & I \end{pmatrix} \frac{\partial^2 H(x,u,t)}{\partial(x,u)^2} \begin{pmatrix} T^{-1} & 0 \\ 0 & I \end{pmatrix}.$$

Suppose moreover that $J - R$ is nonsingular. Then the variational system $\mathrm{d}\Sigma$ of Σ and its adjoint $(\mathrm{d}\Sigma)^*$ of Σ have almost the same state-space realizations.

Remark 1. Suppose the Hessian of the Hamiltonian with respect to (x,u) satisfies

$$\frac{\partial^2 H(x,u,t)}{\partial(x,u)^2}(t - t^0) = \frac{\partial^2 H(x,u,t)}{\partial(x,u)^2}(t^1 - t), \quad {}^\forall t \in [t^0, t^1].$$

Then under appropriate initial conditions of Σ,

$$(\mathrm{d}\Sigma(u))^*(v) \approx \mathcal{R} \circ (\Sigma(u + \mathcal{R}(v)) - \Sigma(u)) \tag{2}$$

holds when v is small where $\mathcal{R}$ is a time-reversal operator defined by $\mathcal{R}(u)(t - t^0) = u(t^1 - t)$ for $\forall t \in [t^0, t^1]$.

Equation (2) implies that we can calculate the input-output mapping of the variational adjoint by only using the input-output data of the original system.

2.2 Optimal control via iterative learning

Let us consider the system Σ in (1) and a cost function $\Gamma : L_2^m[t^0, t^1] \times L_2^r[t^0, t^1] \to \mathbb{R}$ as follows

$$\Gamma(u,y) = \frac{1}{2}\int_{t^0}^{t^1} \Big(u(t)^{\mathrm{T}} \Lambda_u u(t) + (y(t) - y^d(t))^{\mathrm{T}} \Lambda_y (y(t) - y^d(t)) \Big) \mathrm{d}t,$$

where $y^d \in L_2^r[t^0, t^1]$ represents a desired output and $\Lambda_u \in \mathbb{R}^{m\times m}$ and $\Lambda_y \in \mathbb{R}^{r\times r}$ are positive definite matrices. The objective is to find the optimal input minimizing the cost function $\Gamma(u,y)$. Note that the Fréchet derivative of Γ is $\mathrm{d}\Gamma(u,y)$. It follows from well-known Riesz's representation theorem that there exists an operator $\Gamma'(u,y)$ such that

$$\begin{aligned} \mathrm{d}(\Gamma(u,y)) &= \mathrm{d}\Gamma(u,y)(\mathrm{d}u, \mathrm{d}y) \\ &= \langle \Gamma'(u,y), (\mathrm{d}u, \mathrm{d}y) \rangle_{L_2}. \end{aligned}$$

Here we can calculate

$$\begin{aligned} \mathrm{d}(\Gamma(u,y)) &= \langle (\Lambda_u u, \Lambda_y (y - y^d)), (\mathrm{d}u, \mathrm{d}y) \rangle_{L_2} \\ &= \langle \Lambda_u u, \mathrm{d}u \rangle_{L_2} + \langle \Lambda_y (y - y^d), \mathrm{d}\Sigma(u)(\mathrm{d}u) \rangle_{L_2} \\ &= \langle \Lambda_u u + (\mathrm{d}\Sigma(u))^* \Lambda_y (y - y^d), \mathrm{d}u \rangle_{L_2}. \end{aligned}$$

Therefore the steepest descent method implies that we should change the input u such that

$$\mathrm{d}u = -K\big(\Lambda_u u + (\mathrm{d}\Sigma(u))^* \Lambda_y (y - y^d)\big),$$

where K is an appropriate positive gain. Hence the iteration law should be taken as

$$u_{(i+1)} = u_{(i)} - K_{(i)}\big(\Lambda_u u_{(i)} + (\mathrm{d}\Sigma(u))^* \Lambda_y (y_{(i)} - y^d)\big). \tag{3}$$

Here i denotes the i-th iteration in laboratory experiments. Suppose Equation (2) holds, then ILC law based on variational symmetry is given by

$$u_{(2i-1)} = u_{(2i-2)} + \mathcal{R}(\Lambda_y (y_{(2i-2)} - y^d)) \tag{4}$$

$$u_{(2i)} = u_{(2i-2)} - K_{(2i-2)}\big(\Lambda_u u_{(2i-2)} + \mathcal{R}(y_{(2i-1)} - y_{(2i-2)})\big) \tag{5}$$

provided that the initial input $u_{(0)}$ is equivalent to zero.

This pair of iteration laws Equations (4) and (5) implies that this learning procedure needs two steps laboratory experiments. In the ($2i$-1)-th iteration, we can calculate the input and output signals of $(\mathrm{d}\Sigma(u))^*$ by Equation (2) with the output signal of $\Sigma(u+\mathcal{R}(v))$. Input for the $2i$-th iteration is generated by (3) with these signals.

3 Extension of ILC for time derivatives

Let us recall that there is a constraint with respect to cost functions in the iterative learning control method in [9]. For the system Σ in (1), a possible choice of an optimal control type cost function used in the iterative learning control is a functional of u and y, and it is not possible to choose a functional of $\dot{y}$ the time derivative of the output. However, the signal $\dot{y}$ often plays an important role in control of mechanical systems. In particular, it is important to check the behavior of $\dot{y}$ for the gait trajectory generation problem. In this section, we extend the iterative learning control method referred in the previous section to take the time derivative $\dot{y}$ into account.

Let us consider the Hamiltonian system in (1) and suppose that the following assumption holds.

Assumption 1 *The conditions* $\mathrm{d}y(t^0) = 0$ *and* $\mathrm{d}y(t^1) = 0$ *hold.*

In the iterative learning control, it is assumed that all the initial conditions are the same in each laboratory experiment in general. Therefore the condition $\mathrm{d}y(t^0) = 0$ always holds. But the other one $\mathrm{d}y(t^1) = 0$ does not hold in general. In order to let the latter condition $\mathrm{d}y(t^1) = 0$ hold approximately, we can employ an optimal control type cost function such as $\int_{t^1-\epsilon}^{t^1} \|y(t) - y^d(t)\|^2 \mathrm{d}t$ with a small constant $\epsilon > 0$ as in [10].

3.1 Pseudo adjoint of the time derivative operator

Here we investigate a pseudo adjoint of the time derivative operator to take account of the time derivative of the output signal $\dot{y}$ in the iterative learning control procedure.

Consider a differentiable signal $\xi \in L_2[t^0, t^1]$ and a time derivative operator $D(\cdot)$ which maps the signal ξ into its time derivative is defined by

$$D(\xi)(t) := \frac{\mathrm{d}\xi(t)}{\mathrm{d}t}. \tag{6}$$

Let us provide the following lemma to define the pseudo adjoint of the time derivative operator.

Lemma 1. *Consider differentiable signals ξ and $\eta \in L_2[t^0, t^1]$. Suppose that the signal ξ satisfies the condition*

$$\xi(t^0) = \xi(t^1) = 0. \tag{7}$$

Then the following equation holds

$$\langle \eta, D(\xi) \rangle_{L_2} = \langle -D(\eta), \xi \rangle_{L_2}. \tag{8}$$

Proof. Consider the inner product of η and $D(\xi)$. Let us calculate that

$$\langle \eta, D(\xi) \rangle_{L_2} = \int_{t^0}^{t^1} \eta(t)^{\mathrm{T}} \frac{\mathrm{d}\xi(t)}{\mathrm{d}t} \mathrm{d}t = \Big[\eta(t)^{\mathrm{T}} \xi(t)\Big]_{t^0}^{t^1} - \int_{t^0}^{t^1} \frac{\mathrm{d}\eta(t)}{\mathrm{d}t}^{\mathrm{T}} \xi(t) \mathrm{d}t.$$

Since $\xi(t)$ satisfies the condition (7), $\Big[\eta(t)^{\mathrm{T}} \xi(t)\Big]_{t^0}^{t^1} = 0$ holds and we can calculate that

$$\langle \eta, D(\xi) \rangle_{L_2} = -\int_{t^0}^{t^1} \frac{\mathrm{d}\eta(t)}{\mathrm{d}t}^{\mathrm{T}} \xi(t) \mathrm{d}t = \langle -D(\eta), \xi \rangle_{L_2}. \tag{9}$$

Then (9) implies (8). □

This lemma implies

$$D^* = -D$$

for a certain class of input signals.

3.2 Application to iterative learning control

Here we take the following cost function to illustrate the proposed method

$$\Gamma(\dot{y}) = \frac{1}{2} \int_{t^0}^{t^1} \Big((\dot{y}(t) - \dot{y}^d(t))^{\mathrm{T}} \Lambda_{\dot{y}} (\dot{y}(t) - \dot{y}^d(t)) \Big) \mathrm{d}t. \tag{10}$$

Here $\dot{y}^d$ is a differentiable desired velocity which satisfies $\dot{y}^d \in L_2[t^0, t^1]$. Suppose that the output y satisfies Assumption 1. Then we have

$$\mathrm{d}(\Gamma(\dot{y})) = \langle \Lambda_{\dot{y}}(\dot{y} - \dot{y}^d), \mathrm{d}\dot{y} \rangle_{L_2}.$$

The authors' former result [9] can not directly apply to this cost function (10) because it contains $\dot{y}$. Here let us rewrite $\dot{y}$ as $\dot{y} = D(y)$ with the time derivative operator $D(\cdot)$ defined in (6). Then we have

$$\mathrm{d}\dot{y} = \mathrm{d}D(y)(\mathrm{d}y).$$

Note that the time derivative operator is linear, we obtain $\mathrm{d}\dot{y} = D(\mathrm{d}y)$. Using Assumption 1 and (8), we can obtain

$$\begin{aligned} \mathrm{d}(\Gamma(\dot{y})) &= \langle \Lambda_{\dot{y}}(\dot{y} - \dot{y}^d), D(\mathrm{d}y) \rangle_{L_2} \\ &= \langle -D\Big(\Lambda_{\dot{y}}(\dot{y} - \dot{y}^d)\Big), \mathrm{d}y \rangle_{L_2} \\ &= \langle (\mathrm{d}\Sigma(u))^* \Big(-D(\Lambda_{\dot{y}}(\dot{y} - \dot{y}^d))\Big), \mathrm{d}u \rangle_{L_2}. \end{aligned}$$

As we mentioned above, the proposed method allows one to apply the ILC method to cost functions of state variables which do not appear in the output y.

4 Optimal gait generation

In this section, a cost function to generate a symmetric gait is proposed, to which the proposed method in the previous section is applied.

4.1 Description of the plant

Let us consider a passive hopping robot in [7, 8] depicted in Fig. 1. The body and the leg have mass m_b and m_l and moment of inertia J_b and J_u respectively. The distance between the hip joint and the center of mass of the leg is d. Let us define the equivalent leg inertia $J_l := J_u + m_b m_l d^2/(m_b + m_l)$. Let us also define the control force of the leg ρ and the control torque of the hip joint τ. Table 1 shows other notations. Furthermore, some assumptions are assumed on this robot. An important one is as follows. See [7, 8] for the rest of them.

Assumption 2 *The foot does not bounce back nor slip on the ground (inelastic impulsive impact).*

A notion of phases is introduced: the stance phase and the flight phase. When the leg touches the ground the robot is said to be in the stance phase, and when the leg is above the ground it is said to be in the flight phase. The stance time represents the time interval during the stance phase and the flight

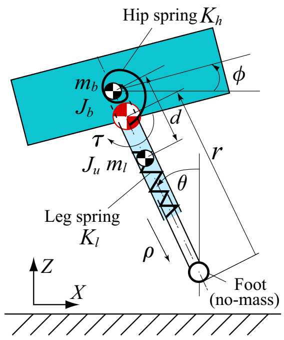

Fig. 1. Description of the plant

Table 1. Notations

Notation	Meaning	Unit
r_0	natural leg length	m
m	total mass	kg
g	gravity acceleration	m/s^2
K_l	leg spring stiffness	kgm^2
K_h	hip spring stiffness	kgm^2
T_s	stance time	s
T_f	flight time	s

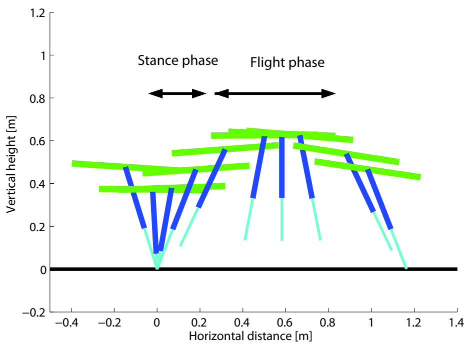

Fig. 2. Subsequent one step of hopping. (The robot moves from left to right.)

time is defined in a similar way. The robot moves between these two phases alternately. See Fig. 2.

In the stance phase, let us define the generalized coordinate q as $q := (r, \theta, \phi)^{\mathrm{T}} \in \mathbb{R}\times\mathbb{S}^1\times\mathbb{S}^1$, the generalized momentum p as $p := (p_r, p_\theta, p_\phi)^{\mathrm{T}} \in \mathbb{R}^3$, input u as $u := (\rho, \tau)^{\mathrm{T}} \in \mathbb{R}^2$ and the inertia matrix $M(q)$ as

$$M(q) := \begin{pmatrix} m & 0 & 0 \\ 0 & J_l + mr^2 & 0 \\ 0 & 0 & J_b \end{pmatrix} \in \mathbb{R}^{3\times 3}.$$

Then, the dynamics is described as a Hamiltonian system in (1) with the Hamiltonian $H(q,p,u)$ represented as

$$\begin{aligned} H(q,p,u) &= \frac{1}{2}p^{\mathrm{T}}M(q)^{-1}p - mgr(1-\cos\theta) \\ &+ \frac{1}{2}K_l(r-r_0)^2 + \frac{1}{2}K_h(\theta-\phi)^2 - u^{\mathrm{T}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \end{pmatrix} q. \end{aligned} \tag{11}$$

Let us consider the dynamics in the flight phase as below

$$\begin{cases} \ddot{x} = 0 \\ \ddot{z} = -g \\ J_l\ddot{\theta} + J_b\ddot{\phi} = 0 \\ J_b\ddot{\phi} = K_h(\theta-\phi) + \tau_f \end{cases}.$$

Here the variables x and z represent the horizontal and vertical positions of the center of mass and τ_f is the control torque.

4.2 Problem setting

This section sets the control problem to get the periodic gait based on [11]. Consider the behavior in a flight phase and let $\mathcal{J}$ denote the map from the initial state to the terminal state in this phase. This map can be regarded as a discrete map which connects two adjacent stance phases. Then the desired map of $\mathcal{J}$ is given by

$$\mathcal{J}_s := (r,\theta,\phi,p_r,p_\theta,p_\phi) \mapsto (r,-\theta,-\phi,-p_r,p_\theta,p_\phi). \tag{12}$$

Let us explain why such a map $\mathcal{J}_s$ in (12) is desired. Let us define the flow Φ_t in the stance phase with no inputs, i.e., $\rho \equiv \tau \equiv 0$ by $\Phi_t := (q(t^0), p(t^0)) \mapsto (q(t^0+t), p(t^0+t))$. For Equations (11) and (12), the Hamiltonian $H(q,p,0)$ is invariant with respect to $\mathcal{J}_s$. Therefore $\mathcal{J}_s$ and Φ_t satisfy $\mathcal{J}_s \circ \Phi_{T_s} = \mathrm{id}$ where id represents the identity mapping. If this equation holds, then a periodic gait is generated.

The leg angle θ is the most important state variable, because it has a direct effect to avoid falling. However, it is difficult to control the variable θ in the stance phase, since this robot has no foot. As in [11], we apply no input in the stance phase, and try to control the variable θ to let $\mathcal{J} = \mathcal{J}_s$ hold in the flight phase.

In [8], dead-beat control is used. Although it works well, it requires the precise knowledge of the plant system. Here we try to use iterative learning control based on variational symmetry with a special cost function given in the following section which will generate an optimal flow in the flight phase without the precise knowledge of the system.

4.3 Application of iterative learning control

Let us define the desired values of θ and $\dot{\theta}$ as follows (we let $t^0 = 0$ for simplicity in what follows)

$$\theta^d := \theta\big|_{t=T_s+T_f} = -\theta\big|_{t=T_s} \tag{13}$$

$$\dot{\theta}^d := \dot{\theta}\big|_{t=T_s+T_f} = \dot{\theta}\big|_{t=T_s}. \tag{14}$$

As for the model mentioned above, energy dissipation occurs at the touchdown. Let E_- and E_+ represent the energies just before the touchdown and just after it. Then ΔE the variation of the energy between them can be calculated as

$$\Delta E := E_- - E_+ = \frac{mJ_l}{2(J_l + mr_0^2)}\mu_-^2 \tag{15}$$

by [8], where μ_- is defined by

$$\mu_- := v_{x_-}\cos\theta_- + v_{z_-}\sin\theta_- + \frac{r_0}{J_l + mr_0^2}p_{\theta_-}.$$

Here v_{x_-} and v_{z_-} represent the horizontal and vertical velocity of the center of mass. If the total mechanical energy is completely preserved, that is, a condition

$$\mu_- = 0 \tag{16}$$

holds at the touchdown, then it is expected that periodic gait trajectories are autonomously generated. In fact, [8] implies that the condition (16) is satisfied if the control objects (13) and (14) are achieved, and if the initial condition is appropriately chosen (the way how to choose is described in [8]).

Now we propose a novel cost function as

$$\Gamma(\theta, \dot{\theta}, u) := \frac{K_\theta}{2}\|\theta - (-\mathcal{R}(\theta))\|_{L_2}^2 + \frac{K_{\dot{\theta}}}{2}\|\dot{\theta} - \mathcal{R}(\dot{\theta})\|_{L_2}^2 + \frac{K_u}{2}\|u\|_{L_2}^2 \tag{17}$$

where K_θ, $K_{\dot{\theta}}$ and K_u represent appropriate positive constants. $\mathcal{R}$ is the time-reversal operator as defined in Sect. 2. The first and the second terms in the right hand side of Equation (17) are expected to make Assumption 1 approximately hold. It is expected that we can generate an optimal trajectory satisfying Equations (13) and (14) which minimizes the L_2 norm of the control input. Furthermore, there is no energy transfer except for the control input.

Let us recall the fact that gait trajectories are essentially periodic. However, ILC can not generate periodic trajectories. Let us connect the stance flow Φ_t and that generated by the cost function (17). Take the connected trajectory as a single period of a periodic gait trajectory. Therefore if Equation (16) holds, then we can generate the optimal periodic trajectory.

Now, let us define the input $u = \tau_f$ and the output $y = \theta$. We can calculate the iteration law as in (3) and in (11) (Derivation of the law is omitted due to limitations of space)

$$u_{(2i-1)} = u_{(2i-2)} + \mathcal{R}\Big(2K_\theta(\mathrm{id}+\mathcal{R})(y_{(2i-2)}) - 2K_{\dot\theta}(\mathrm{id}-\mathcal{R})(\ddot{y}_{(2i-2)})\Big) \quad (18)$$

$$u_{(2i)} = u_{(2i-2)} - K_{(2i-2)}\Big(K_u u_{(2i-2)} + \mathcal{R}\big(y_{(2i-1)} - y_{(2i-2)}\big)\Big). \quad (19)$$

5 Simulation

We apply the proposed iteration law in (18) and (19) to the hopping robot introduced in the previous section. We proceed 50 steps of the learning algorithm, that is, we execute 100 simulations.

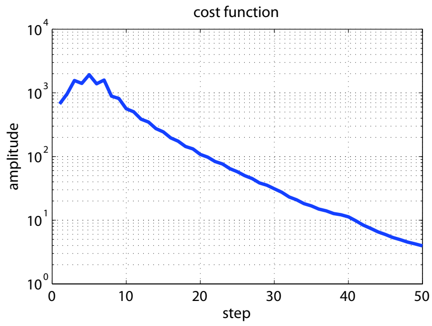

Fig. 3. Cost function

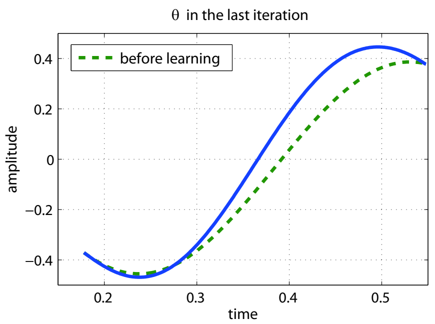

Fig. 4. Response of θ

Fig. 3 shows that the cost function (17) almost decreases at each experiment. This implies that the output trajectory converges to the optimal one smoothly. Since we choose the initial input $u_{(0)}(t) \equiv 0$, Assumption 1 does not hold in the beginning. This is probably the reason why the cost function

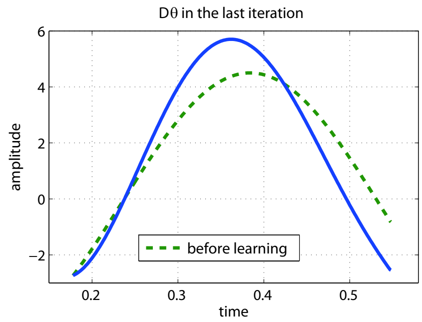

Fig. 5. Response of $\dot{\theta}$

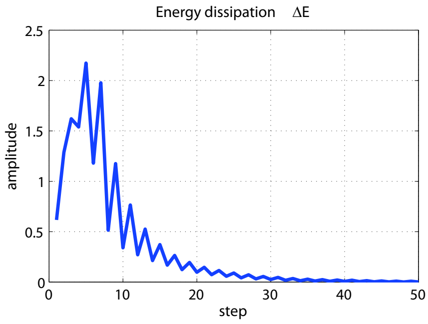

Fig. 6. Variation of the Energy $\varDelta E$

increases in the beginning of the learning procedure. In Figs. 4 and 5, the solid lines show the responses of θ and $\dot{\theta}$ at the last step and the dotted lines depict the initial trajectories corresponding to 0 input. These figures show that both θ and $\dot{\theta}$ converge to the trajectories satisfying Equations (13) and (14). Furthermore, Fig. 6 exhibits that the variation of the energy at the touchdown $\varDelta E$ in (15) converges to zero automatically.

6 Conclusion

In this paper, we have proposed an extension of the iterative learning control based on variational symmetry employing a pseudo adjoint of the time derivative operator. This allows one to execute iterative learning with optimal control type cost function including time derivatives of the output signals. Application of this method to gait generation problem for a hopping robot derives an optimal gait trajectories without using precise knowledge of the plant. Finally, numerical simulations have demonstrated the effectiveness of the proposed method.

Although we succeed in generating optimal gait trajectories minimizing the L_2 norm of the control input, they are *not optimal* in a sense that the hopping gaits with zero input, *passive gaits*, are not obtained. This is because the algorithm proposed in this paper can not take into account the variation of the initial conditions. However passive gaits depend on their initial conditions. It means that only the input iteration can not generate such gaits. In our recent result in [12], we propose a novel algorithm to generate *passive gaits* by employing an update law for the initial conditions as well as that for the feedforward input.

The proposed cost function can not deal with asymmetric gaits. Our another result in [13], a certain class of non-symmetric periodic gaits are considered.

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