On repetitive control of Hamiltonian systems based on variational symmetry

Kenji Fujimoto and Satoshi Satoh

Abstract— **This paper is concerned with repetitive control of Hamiltonian systems which is based on iterative learning control utilizing the variational symmetry of those systems. Variational symmetry allows us to obtain an algorithm to solve a certain class of optimal control problems in the repetitive control framework. A convergence analysis of this algorithm is also discussed. Furthermore, some simulations demonstrate the effectiveness of the proposed method.**

Keywords— **Nonlinear control, Hamiltonian systems, Iterative learning control, Repetitive control**

I. Introduction

Iterative learning control method proposed in [1] is an algorithm to generate a feedforward input achieving a trajectory tracking control (on a finite time interval) without using the precise information of the plant system. Since this algorithm does not require the precise model of the plant, it is robust against modeling errors and many researchers worked on this topic, e.g. [2], [9]. So far, however, this algorithm was only applicable to trajectory tracking control problems. Recently, the authors proposed a novel iterative learning control method based on variational symmetry of Hamiltonian systems [7], [8], [4].

In this result, it is proved that the variational systems of Hamiltonian systems are symmetric and this property can be utilized for executing the iterative algorithm for optimal control problems without using the precise information of the plant.

On the other hand, in the linear control systems theory, repetitive control is also a useful tool which has a close relationship to iterative learning control, e.g. [11], [10]. Repetitive control is also a kind of a learning method for a trajectory tracking control problem with time periodic reference trajectories without using precise information of the plant. Only the difference between repetitive control and iterative learning control is the type of the reference trajectories: time periodic one (with the infinite length) and one on a finite time interval. However, the repetitive control for optimal control problem, as in the iterative leaning control case, was not investigated so far.

The present paper focuses on repetitive control of Hamiltonian systems and proposed a new repetitive control framework based on variational symmetry. Although this approach has a defect that the reference (or desired) trajectories have to be time-symmetric with respect to its period, it also has a big advantage that it is applicable to a class of optimal control problems as well as conventional trajectory tacking control ones. We also provide a convergence analysis which guarantees the convergence of the output trajectory on the reference (or desired) one under certain conditions. Furthermore, numerical simulations of a robot manipulator demonstrate the effectiveness of the proposed method.

K. Fujimoto is with the Department of Mechanical Science and Engineering, Graduate School of Engineering, Nagoya University, Furo-cho, Chikusa-ku, Nagoya 464-8603, Japan `k.fujimoto@ieee.org`

S. Satoh is with the Department of Mechanical Science and Engineering, Graduate School of Engineering, Nagoya University, Furo-cho, Chikusa-ku, Nagoya 464-8603, Japan `s_satou@nuem.nagoya-u.ac.jp`

II. Iterative learning optimal control of Hamiltonian systems based on variational symmetry

This section briefly refers to some preliminary backgrounds.

A. Variational symmetry

The plant system considered here is a Hamiltonian system with dissipation Σ with a controlled Hamiltonian $H(x,u,t)$ as $(x^1, y) = \Sigma(x^0, u)$:

$$\left\{ \begin{array}{rcl} \dot{x} &=& (J-R)\dfrac{\partial H(x,u,t)}{\partial x}^{\mathrm{T}}, \quad x(t^0) = x^0 \\ y &=& -\dfrac{\partial H(x,u,t)}{\partial u}^{\mathrm{T}} \\ x^1 &=& x(t^1) \end{array} \right. . \quad (1)$$

Here the structure matrix $J \in \mathbb{R}^{n\times n}$ and the dissipation matrix $R \in \mathbb{R}^{n\times n}$ are skew-symmetric and symmetric positive semi-definite, respectively. The matrix R represents dissipative elements such as friction of mechanical systems and resistance of electric circuits. For this system, the following theorem holds.

Theorem 1: [8] Consider the Hamiltonian system with dissipation and the controlled Hamiltonian Σ in (1). Suppose that J and R are constant and that there exists a nonsingular matrix $T \in \mathbb{R}^{n\times n}$ satisfying

$$\begin{array}{rl} J &= -TJ\,T^{-1} \qquad R = TR\,T^{-1} \\ \dfrac{\partial^2 H(x,u,t)}{\partial (x,u)^2} &= \begin{pmatrix} T & 0 \\ 0 & I \end{pmatrix} \dfrac{\partial^2 H(x,u,t)}{\partial (x,u)^2} \begin{pmatrix} T & 0 \\ 0 & I \end{pmatrix}^{-1} . \end{array} \quad (2)$$

Then the Fréchet derivative of Σ is described by another linear Hamiltonian system $(x_v^1, y_v) =$

$\mathrm{d}\Sigma((x^0,u),(x_v^0,u_v))$:

$$\begin{cases} \dot{x} &= (J-R)\dfrac{\partial H(x,u,t)}{\partial x}^{\mathrm{T}}, \qquad\qquad x(t^0)=x^0 \\ \dot{x}_v &= (J-R)\dfrac{\partial H_v(x,u,x_v,u_v,t)}{\partial x_v}^{\mathrm{T}}, \quad x_v(t^0)=x_v^0 \\ y_v &= -\dfrac{\partial H_v(x,u,x_v,u_v,t)}{\partial u_v}^{\mathrm{T}} \\ x_v^1 &= x_v(t^1) \end{cases}$$

with a controlled Hamiltonian $H_v(x,u,x_v,u_v,t)$

$$H_v(x,u,x_v,u_v,t)=\frac{1}{2}\begin{pmatrix} x_v \\ u_v \end{pmatrix}^{\mathrm{T}}\frac{\partial^2 H(x,u,t)}{\partial(x,u)^2}\begin{pmatrix} x_v \\ u_v \end{pmatrix}.$$

Furthermore, the adjoint of the variational system with zero initial state $u_a \mapsto y_a = (\mathrm{d}\Sigma^{x^0}(u))^*(u_a)$ *is given by*

$$\begin{cases} \dot{x} &= (J-R)\dfrac{\partial H(x,u,t)}{\partial x}^{\mathrm{T}} \\ \dot{\bar{x}}_v &= -(J-R)\dfrac{\partial H_v(x,u,\bar{x}_v,u_a,t)}{\partial \bar{x}_v}^{\mathrm{T}} \\ y_a &= -\dfrac{\partial H_v(x,u,\bar{x}_v,u_a,t)}{\partial u_a}^{\mathrm{T}} \end{cases} \tag{3}$$

with the initial state $x(t^0)=x^0$ *and the terminal state* $\bar{x}_v(t^1)=0$*. Suppose moreover that* $J-R$ *is nonsingular. Then the adjoint* $(x_a^1,u_a)\mapsto(x_a^0,y_a)=(\mathrm{d}\Sigma(x^0,u))^*(x_a^1,u_a)$ *is given by the same state-space realization (3) with the initial sate* $x(t^0)=x^0$*, the terminal state* $\bar{x}_v(t^1)=-(J-R)T\ x_a^1$ *and* $x_a^0=-T^{-1}(J-R)^{-1}\bar{x}_v(t^0)$.

This theorem reveals that the variational system and its adjoint of a Hamiltonian system in the form (1) have almost the same state-space realizations. This means that the input-output mapping of the adjoint can be produced by the input-output data of the original Hamiltonian system as

$$\mathcal{R}\circ(\mathrm{d}\Sigma(u))^*\circ\mathcal{R}(v)=\mathrm{d}\Sigma(\bar{u})(v)\approx\Sigma(\bar{u}+v)-\Sigma(\bar{u}) \tag{4}$$

provided appropriate boundary conditions are selected, where $\mathcal{R}$ is the time reversal operator defined by

$$\mathcal{R}(u)(t-t^0)=u(t^1-t),\quad \forall t\in[t^0,t^1]. \tag{5}$$

This property is utilized for solving the optimal control problems in which the adjoint operator plays an important role.

Remark 1: It is noted that if the system is a *gradient system* [3] which is a nonlinear generalization of a linear symmetric system, that is, $J=0$, then the assumption (2) in Theorem 1 is automatically satisfied with $T=I$. On the other hand, if the system is conservative, that is, $R=0$ then it is self-adjoint in the usual sense [8].

B. Optimal control via iterative learning

Let us consider the system $\Sigma: U\to Y$ in (1) and a cost function $\Gamma: X^2\times U\times Y\to\mathbb{R}$. The objective is to find the optimal input $(x_\star^0,u_\star)$ minimizing the cost function $\Gamma(x^0,u,x^1,y)$. In general, however, it is difficult to obtain a global minimum since the cost function Γ is not convex. Hence we try to obtain a local minimum here. Here we can calculate

$$\begin{aligned} &\mathrm{d}\left(\Gamma((x^0,u),\Sigma(x^0,u))\right)(\mathrm{d}x^0,\mathrm{d}u) \\ &=\mathrm{d}\Gamma((x^0,u),\Sigma(x^0,u))\left((\mathrm{d}x^0,\mathrm{d}u),\mathrm{d}\Sigma(x^0,u)(\mathrm{d}x^0,\mathrm{d}u)\right) \\ &=\langle\Gamma'((x^0,u),\Sigma(x^0,u)),\begin{pmatrix}\mathrm{id}_{X\times U}\\ \mathrm{d}\Sigma(x^0,u)\end{pmatrix}(\mathrm{d}x^0,\mathrm{d}u)\rangle_{X^2\times U\times Y} \\ &=\langle\left(\mathrm{id}_{X\times U},(\mathrm{d}\Sigma(x^0,u))^*\right)\Gamma'(x^0,u,x^1,y),(\mathrm{d}x^0,\mathrm{d}u)\rangle_{X\times U}. \end{aligned}$$

Therefore, if the adjoint $(\mathrm{d}\Sigma(x^0,u))^*$ is available, we can reduce the cost function Γ down at least to a local minimum by an iteration law with a $K_{(i)}>0$.

$$\begin{aligned} u_{(i+1)} &= u_{(i)}-K_{(i)}\,(0_{UX},\mathrm{id}_U) \\ &\left(\mathrm{id}_{X\times U},\ (\mathrm{d}\Sigma(x_{(i)}^0,u_{(i)}))^*\right)\Gamma'(x_{(i)}^0,u_{(i)},x_{(i)}^1,y_{(i)}) \end{aligned} \tag{6}$$

The results in the previous section enable one to execute this procedure without using the parameters of the original operator Σ by the relation (4), provided Σ is a Hamiltonian system and the boundary conditions are selected appropriately. In [8], this framework is effectively utilized for iterative learning control (of trajectory tracking) for a 'round trip' type trajectory. More precise discussion for optimal control will be made in the following sections.

III. MAIN RESULTS

In this section, we propose a new algorithm which can solve a class of repetitive control problems based on variational symmetry of Hamiltonian systems.

A. Repetitive control

A typical repetitive control problem is to achieve trajectory tracking control for a periodic reference trajectory by learning (experiments) without using precise information of the plant [11]. A conventional iterative learning control is also to achieve trajectory tracking control by learning but the reference trajectory is defined on a finite time interval, that is, tracking is achieved by several experiments with finite time interval (with the same initial states) [1].

The approach taken here is to provide a repetitive control framework using the iterative learning control based on variational symmetry. Variational symmetry allows us to obtain an algorithm to solve a certain class of optimal control problems in the repetitive control framework. Here we adopt the following strategy: Suppose that the plant Hamiltonian system is controlled by a feedback designed by generalized canonical transformations [6] so that the closed loop system is again described by a Hamiltonian system (1). See Section IV for a concrete example of constructing such a control system and refer to [8] for the detail. Suppose also that the desired reference trajectory or the desired optimal trajectory is L-periodic and time-symmetric[1], and that it contains a stationary point. Then

[1] Since the iterative learning control based on variational symmetry needs experiments with time-reversal trajectories with respect to the trajectory to be learned, the reference or desired trajectory has to be time-symmetric in the repetitive control framework.

apply the iterative learning control for the first period L from the stationary point and then wait for the state to converge on the stationary point. When the state approaches sufficiently close to the stationary point, the next iterative learning control with time period L starts and continue in the same manner. A more detailed procedure is explained below. (See Figure 1 as well.)

Step 0:Suppose that the initial state $x(0)$ is a stationary state. Set $t^0_{(1)} := 0$ and goto Step 1.

Step i:(a) Set $t^L_{(i)} := t^0_{(i)} + L$ and apply the iterative learning control procedure stated in Section II for the time $t \in [t^0_{(i)}, t^L_{(i)}]$.

(b) Set $u(t) \equiv 0$ for $t \geq t^L_{(i)}$ and find the smallest $\tau \geq t^L_{(i)}$ satisfying

$$\|x(\tau)\| \leq b \tag{7}$$

with a prescribed constant $b > 0$, and $u(t) \equiv 0$ is applied for $t \in [t^L_{(i)}, \tau]$. Define $\Delta t_{(i)} := \tau - t^L_{(i)}$ and $t^0_{(i+1)} := \tau$ and go to Step $i+1$.

By this algorithm, an adjustment parameter b with an excess converging time $\Delta t_{(i)}$ is introduced, so the initial state of iterative learning control in each period is sufficiently close to the stationary point $x(0)$. However if $\Delta t_{(i)}$ converges on 0 as i grows, then we can conclude that we asymptotically obtain a repetitive control system. So the following section discusses the behavior of the excess converging time $\Delta t_{(i)}$.

B. Convergence analysis

In the procedure proposed in the previous section, we have introduced an adjustment parameter b in Equation (7) and, consequently, we need to employ the corresponding excess converging time $\Delta t_{(i)}$ which is 0 in the conventional repetitive control. This section discusses when the converging time $\Delta t_{(i)}$ becomes (converges on) 0.

Under certain smoothness assumptions on the system (1) and the cost function Γ, we can prove the following theorems.

Theorem 2: There exists a constant $b_{\max} > 0$ such that for any positive $b \leq b_{\max}$ there exists $\Delta t^b_\infty \geq 0$ satisfying

$$\lim_{i\to\infty} \Delta t_{(i)} = \Delta t^b_\infty,$$

and the state $x(t)$ will converge on a $(L + \Delta t^b_\infty)$-periodic trajectory.

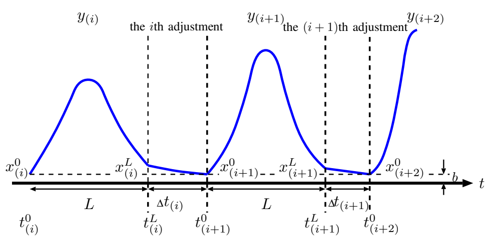

Fig. 1. Typical time response of the proposed repetitive control procedure

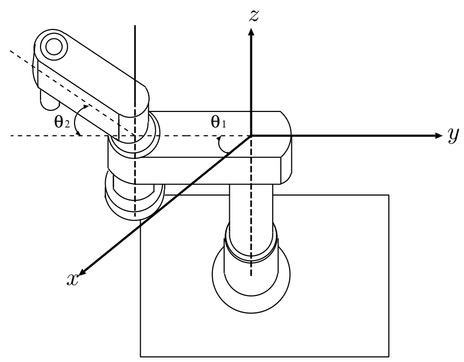

Fig. 2. 2-link robot manipulator

TABLE I

PHYSICAL PARAMETERS

θ_i	the joint angle of the i-th link	[rad]
m_i	the mas of the i-th link	[kg]
l_i	the length of the i-th link	[m]
l_{gi}	the length to the center of gravity	[m]
I_i	the mass of the i-th link	[kgm^2]
d_i	the friction coefficient of the i-th link	[Nms/rad]

Its proof is based on Gronwall-Bellman Lemma and Morse Lemma. It is omitted due to limitation of space. See [5] for the detail.

Here we can prove that the state trajectory always converges on a periodic trajectory. Furthermore, the following theorem guarantees that the excess converging time Δt^b_∞ also converges on 0 as b becomes smaller.

Theorem 3: Suppose that iterative learning control applied to the plant achieves a trajectory tracking control[2]*. Then the following equation holds.*

$$\lim_{b\to 0} \Delta t^b_\infty = 0$$

The proof of this theorem is also omitted here. See [5] for the detail. This theorem implies that the proposed method converges on conventional repetitive control as the adjustment parameter $b \to 0$. This fact allows us to obtain optimal control periodic solutions by repetitive control framework.

IV. SIMULATION

This section exhibits the effectiveness of the proposed method via numerical simulations. Here let us consider a two-link robot manipulator moving on a horizontal plane depicted in Figure 2. As in the figure, the joint angles of the first and the second links are denoted by θ_1 and θ_2, respectively. The physical parameters of this apparatus are summarized in Table I.

Then the dynamics of this apparatus is described by a

[2]For example, the authors' former paper [8] proves that iterative learning control for a trajectory tracking problem of simple mechanical systems always achieves the global minimum.

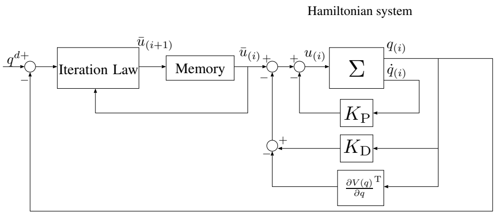

Fig. 3. Iterative learning control of mechanical systems

Hamiltonian control system (1) with

$$
\begin{aligned}
x &= (q, p) \\
q &= (\theta_1, \theta_2) \\
p &= M(q)\dot{q} \\
H(x) &= \frac{1}{2}p^{\mathrm{T}}M(q)^{-1}p - q^{\mathrm{T}}u \\
J &= \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix} \\
R &= \begin{pmatrix} 0 & 0 \\ 0 & \mathrm{diag}(d_1, d_2) \end{pmatrix}
\end{aligned}
$$

and

$$
M(q) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & b_1 + b_2 + 2b_3\cos\theta_2 & b_2 + b_3\cos\theta_2 \\ 0 & 0 & b_2 + b_3\cos\theta_2 & b_2 \end{pmatrix}
$$

where

$$
\begin{aligned}
b_1 &:= I_1 + m_1 l_{g2}^2 + m_2 l_1^2 = \frac{4}{3}m_1 l_{g2}^2 + m_2 l_1^2 \\
b_2 &:= I_2 + m_2 l_{g2}^2 = \frac{4}{3}m_2 l_{g2}^2 \\
b_3 &:= l_1 m_2 l_{g2}.
\end{aligned}
$$

The concrete parameters used in the simulations are $b_1 = 2.292$, $b_2 = 0.600$ and $b_3 = 0.750$. See [8] for the detail of this apparatus.

As explained in Section III, we need to apply a local feedback designed by a generalized canonical transformation in order to obtain the closed loop system to be a Hamiltonian system. Here we employ a PD pre-feedback

$$
u = \bar{u} - K_{\mathrm{P}}q - K_{\mathrm{D}}\dot{q} + \frac{\partial V(q)}{\partial q}^{\mathrm{T}}
$$

with a new input $\bar{u}$ and the PD gains K_{P} and K_{D}. A scalar function $V(q)$ denotes the potential energy of the system. Then the dynamics of the closed-loop system is again described by a Hamiltonian system (1) and this control system is depicted in Figure 3. For this system the output signal is $y = q = (\theta_1, \theta_2)$ describing the joint angles.

A. Trajectory tracking control

Here we consider repetitive control for a trajectory tracking control problem. The desired trajectory for the output $y^d = (\theta_1^d, \theta_2^d)$ is given by

$$
\begin{aligned}
\theta_1^d(t) &:= \frac{1}{2}\sin\frac{\pi}{2}(t-1) + \frac{1}{2} \\
\theta_2^d(t) &:= \begin{cases} 0 & (0 \le t < \frac{3}{8}L) \\ \frac{1}{2}\sin\frac{\pi}{2}(t - 1 + \frac{3}{8}L) + \frac{1}{2} & (t \ge \frac{3}{8}L) \end{cases}
\end{aligned}
$$

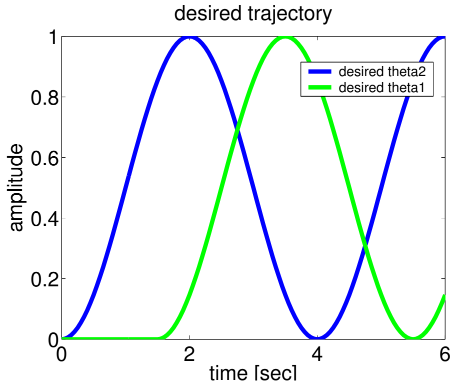

Fig. 4. Desired trajectory

with the period $L = 4$ [s] which is depicted in Figure 4. Notice that θ_1 and θ_2 do not have stationary points simultaneously.

Now apply the proposed method with the following cost function

$$
\Gamma(y) = \frac{1}{2}\|y - y^d\|_{L_2}^2. \qquad (8)
$$

Then the corresponding iterative learning law can be obtained by

$$
\begin{aligned}
\bar{u}_{(2i+1)} &= \bar{u}_{(2i)} + \mathcal{R}\{(y_{(2i)} - y^d)\} \\
\bar{u}_{(2i+2)} &= \bar{u}_{(2i)} - K_{(i)}\mathcal{R}(y_{(2i+1)} - y_{(2i)}).
\end{aligned} \qquad (9)
$$

The simulation results of the repetitive control are depicted in Figures 5–8. Figure 5 denotes the responses of the joint angles θ_1 and θ_2 from the 1st to 8th periods. Figure 6 denotes those with their reference signals in the 55th period. Both figures show that the joint angles are approaching to their desired trajectories. Figure 7 denotes the history of the cost function Γ in Equation (8) with respect to the period i (learning step). This figure shows that the joint angles converge on their reference trajectories, since the cost function monotonically decreases. Furthermore, Figure 8 denotes the excess converging time $\Delta t_{(i)}$ with respect to the period i. This figure shows that the period $(L + \Delta t_{(i)})$ converges on 4 [s] after the 58 period. Thus the proposed repetitive control method works well with a trajectory tracking control problem.

B. Optimal control problem

Next we consider repetitive control for an optimal control problem, that is, a trajectory generation problem. As explained in Footnote 1, the desired (generated) trajectory has to be time-symmetric with respect to the period

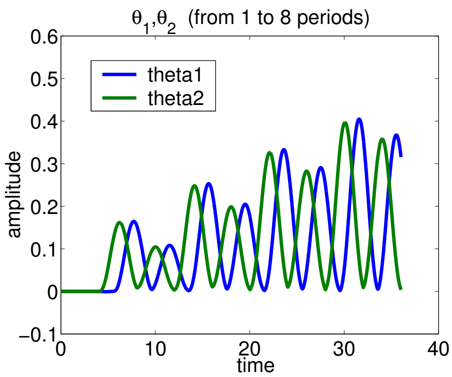

Fig. 5. Responses of θ_1 and θ_2 (from the 1st to 8th periods)

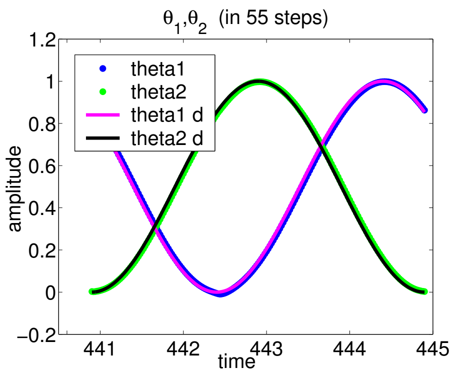

Fig. 6. Responses of θ_1 and θ_2 (in the 55 period)

L^3. Therefore we employ a special cost function which achieves desired intermediate and terminal joint angles with suppressing the input signal u and preserving the time-symmetry of the trajectory

$$\begin{aligned}\Gamma(y) \quad &= \quad \frac{1}{2}k_d\|F(t)(y-y^d)\|_{L_2}^2 + \frac{1}{2}k_t\|y-\mathcal{R}(y)\|_{L_2}^2 \\ &+ \quad \frac{1}{2}k_u\|u\|_{L_2}^2 \end{aligned} \tag{10}$$

[3]More precisely, the Hessian of the Hamiltonian function has to be so [8].

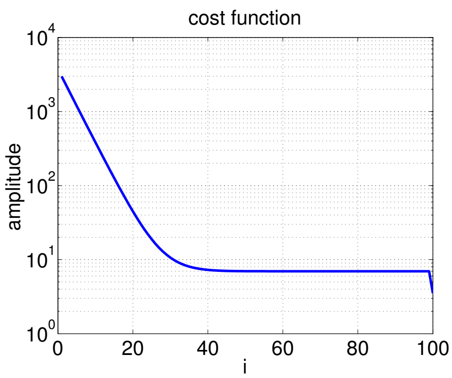

Fig. 7. Cost function Γ for trajectory tracking control

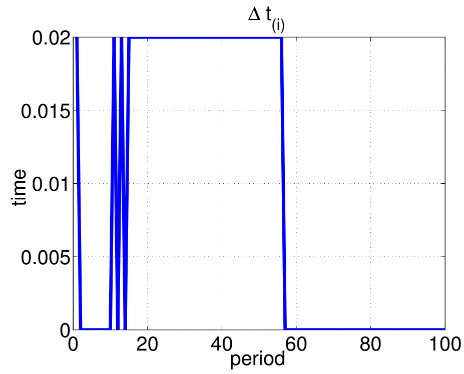

Fig. 8. Excess converging time $\Delta t_{(i)}$ for trajectory tracking control

where k_d, k_t and k_u are positive constants and the function $F(t)$ is defined by

$$F(t) = \begin{cases} 1 & t \in [t_i - \epsilon, t_i + \epsilon] \\ & \qquad (i = 1, 2, \ldots n) \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

with a small constant $\epsilon > 0$. Here the first term of the cost function Γ in Equation (10) is for achieving the desired intermediate and terminal outputs (y^d is an appropriate 'virtual' time-symmetric reference output which is only valid for t satisfying $F(t) \neq 0$), the second for suppressing the input, and the last for time-symmetry. Then the corresponding iterative control law is given as follows.

$$\begin{aligned} \bar{u}_{(2i+1)} &= \bar{u}_{(2i)} + \mathcal{R}\{k_d F(t)(y_{(2i)} - y^d) \\ &+ \quad 2k_t(y_{(2i)} - \mathcal{R}(y_{(2i)}))\} \\ \bar{u}_{(2i+2)} &= \bar{u}_{(2i)}(\mathrm{I_d} - k_u K_{(i)}) \\ &- \quad K_{(i)}\mathcal{R}(y_{(2i+1)} - y_{(2i)}) \end{aligned} \tag{12}$$

where $\mathrm{I_d}$ is the identity. Let the period be $L = 4$ [s], the initial state to be $(0, 0)$, the desired terminal state to be the same as the initial, and the desired intermediate state to be $y^d(L/2) = (\theta_1^d(L/2), \theta_2^d(L/2)) = (1.0, 1.0)$. The simulation results are depicted in Figures 9–11. For the reason of space, the responses with respect the first joint are shown. Figure 9 denotes the time response of the angle of the first joint θ_1 in the 1st, 2nd, 5th, 10th and 100th periods. Figure 10 denotes the history of the cost function Γ in Equation (10). The cost function monotonically decreases and the desired trajectory is generated automatically. Furthermore, Figure 11 denotes the history of the excess converging time $\Delta t_{(i)}$ with respect to the period i. It becomes quite small eventually. Thus repetitive control for an optimal control problem also works well. These simulations demonstrate the effectiveness of the proposed algorithm.

V. Conclusion

In this paper, we have proposed a new framework for repetitive control of Hamiltonian systems based on iterative learning control proposed by the authors previously. Since

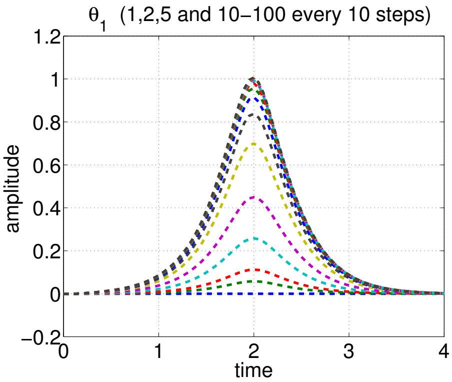

Fig. 9. Response of θ_1 (in the 1st, 2nd, 5th, 10th, 20th, ... 100th periods)

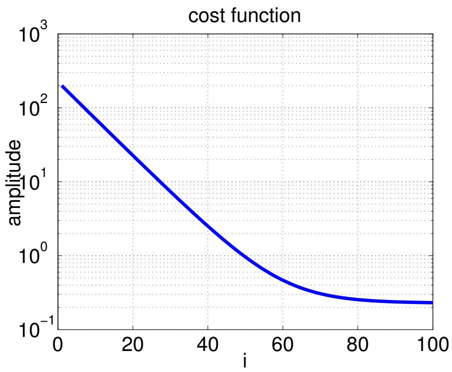

Fig. 10. Cost function Γ for optimal control

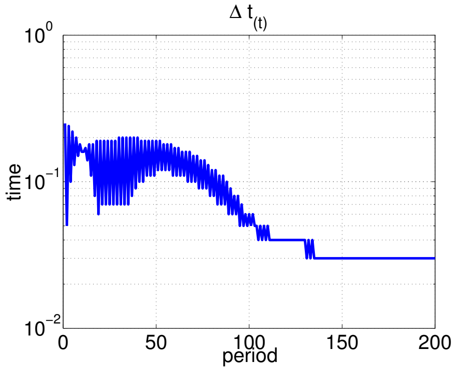

Fig. 11. Excess converging time $\Delta t_{(i)}$ for optimal control

this algorithm is based on variational symmetry of Hamiltonian control systems, it is applicable to certain optimal control problems as well as a conventional trajectory tracking control one. We have shown that the convergence analysis which guarantees the convergence of the trajectory generated by this procedure on the periodic reference or desired trajectory under certain conditions. Furthermore, numerical simulations of a robot manipulator have shown the effectiveness of the proposed method.

We should notice that this result is applicable to gait generation of a hopping robot [12] where the obtained gait is optimal in a sense that it minimizes the input energy [13]. Thus the proposed method is expected to be useful for several purposes.

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