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RESEARCH ARTICLE

Bounded stabilization of stochastic port-Hamiltonian systems

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This paper proposes a stochastic bounded stabilization method for a class of stochastic port-Hamiltonian systems. Both full-actuated and underactuated mechanical systems in the presence of noise are considered in this class. The proposed method gives conditions for the controller gain and design parameters under which the state remains bounded in probability. The bounded region and achieving probability are both assignable, and a stochastic Lyapunov function is explicitly provided based on a Hamiltonian structure. Although many conventional stabilization methods assume that the noise vanishes at the origin, the proposed method is applicable to systems under persistent disturbances.

Keywords: stochastic stability; stochastic Hamiltonian systems; bounded stability; nonlinear stochastic control

1 Introduction

Since there possibly exist uncertainties in controlling dynamical plants, such as noise, disturbance, modeling errors, etc., the stabilization of nonlinear stochastic systems has been studied by many researchers. The literature in Florchinger (1997) deals with a stochastic version of the control Lyapunov function approach for a class of input-affine nonlinear stochastic systems. It provides a sufficient condition for asymptotic stabilizability in probability. In Deng and Krstić (1999), a stochastic output feedback stabilization controller based on the backstepping technique is proposed for strict feedback systems. The notion of stochastic passivity is introduced in Florchinger (1999). As the deterministic passivity-based control (Byrnes et al. (1991)), asymptotic stability in probability can be achieved for stochastic nonlinear systems by the unity feedback of the passive output. We have introduced stochastic port-Hamiltonian systems (SPHSs) in Satoh and Fujimoto (2013) as an extension of deterministic port-Hamiltonian systems (Maschke and van der Schaft (1992)). SPHSs can represent practically important systems with uncertainties such as physical systems, passive electrical networks and nonholonomic systems in the presence of noise. A systematic stabilization method for SPHSs has also been proposed by the authors. This method is based on the stochastic passivity and the stochastic generalized canonical transformation equipped in Satoh and Fujimoto (2013), which is a pair of coordinate and feedback transformations preserving the SPHS structure, and is a stochastic version of the transformation proposed in Fujimoto and Sugie (2001).

However, many stabilization methods including the above ones assume that the noise vanishes at the origin. Stochastic bounded stability, e.g. Kushner (1967), Thygesen (1997), Liu and Raffoul (2009), is a useful concept for a system under persistent disturbances. The literature in Liu and Raffoul (2009) considers an autonomous stochastic system, and provides a sufficient condition of

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a stochastic Lyapunov function such that the expectation of the state can be bounded. Besides, the literatures in Tsinias (1998), Deng et al. (2001), Liu et al. (2008) introduced some variations of the stochastic input-to-state stability, which is an adaptation of the input-to-state stability concept in the deterministic control theory (Sontag and Wang (1995)) to the stochastic one. They provide sufficient conditions of stochastic Lyapunov functions such that the state can be bounded under a deterministic external input, an unknown noise covariance and a stochastic external input, respectively.

On the other hand, in this paper, we consider bounded stabilization of a class of SPHSs based on the bounded stability concept, called (Q_0, Q_1, ρ)-stability, introduced by Kushner (1967). We consider a broader class of mechanical systems in the presence of noise, which includes both full-actuated and underactuated systems. We consider a system with the control input and the system noise with known covariant matrix (normalized as the identity matrix). This setting is more flexible compared to Liu and Raffoul (2009) due to the control input, but is restricted compared to Tsinias (1998), Deng et al. (2001), Liu et al. (2008) due to the absence of external forces. We derive conditions for the controller gain and some design parameters under which the state remains bounded in probability for a given probability and bounds of the state. It is the first advantage of the proposed method compared to the aforementioned methods that both a bounded region for the state and its achieving probability can be assignable. In passivity-based control of a deterministic Hamiltonian system, an energy-based Lyapunov function is often used. Since, however, the time variation of the energy-based Lyapunov function depends only on a part of the state, boundedness of the state cannot be guaranteed in the case of SPHSs with noise which does not vanish at the origin. To solve this problem, we first equip a specific stochastic Lyapunov function based on a structure of a stochastic mechanical system, and then analyze the boundedness of the state. Since the time variation of the proposed Lyapunov function involves all of the state variables, it enables us to evaluate the probability for the state remaining in the specified region based on the martingale theory. Although the aforementioned methods show stability conditions of stochastic Lyapunov functions, they do not provide concrete construction of those functions. The second advantage of the proposed method is that we give not only a bounded stability condition, but also a construction method of a stochastic Lyapunov function.

This paper grew out of our previous report in Satoh and Saeki (2012). The main results here enable one to newly apply the proposed stochastic bounded stabilization method to underactuated mechanical systems.

2 Preliminaries

We consider a class of SPHSs in Satoh and Fujimoto (2013), which is described by the following Itô stochastic differential equation:

$$\begin{cases} \begin{pmatrix} \mathrm{d}q \\ \mathrm{d}p \end{pmatrix} = \begin{pmatrix} 0 & J_1(q,p) \\ -J_1(q,p)^\top & J_2(q,p) - D(q,p) \end{pmatrix} \begin{pmatrix} \frac{\partial H(q,p)}{\partial q}^\top \\ \frac{\partial H(q,p)}{\partial p}^\top \end{pmatrix} \mathrm{d}t \\ \qquad + \begin{pmatrix} 0 \\ G(q) \end{pmatrix} u \, \mathrm{d}t + \begin{pmatrix} 0 \\ \Xi(q,p) \end{pmatrix} \mathrm{d}w, \\ y = G(q)^\top \frac{\partial H(q,p)}{\partial p}^\top = G(q)^\top M(q)^{-1} p \end{cases} \tag{1}$$

with the Hamiltonian $H(q,p) = \frac{1}{2} p^\top M(q)^{-1} p + U(q)$, where $q, p \in \mathbb{R}^m$ are the generalized coordinate and momentum, respectively. A symmetric positive-definite matrix $M(q) \in \mathbb{R}^{m \times m}$ denotes the inertia matrix, and a scalar function $U(q)$ denotes a potential energy, which is assumed to be a sufficiently differentiable positive-definite function. We have proposed a way to assign a proper potential energy to an SPHS in Satoh and Fujimoto (2013), and the literatures in Fujimoto and Sugie (2001), Ortega et al. (2002a,b) for the deterministic Hamiltonian systems are also useful. A positive-semidefinite matrix $D(q,p) \in \mathbb{R}^{m\times m}$ denotes the viscous friction coefficients, and $J_1(q,p) \in \mathbb{R}^{m\times m}$ and $J_2(q,p) \in \mathbb{R}^{m\times m}$ denote the interconnection structure of the system, where $J_1(q,p)$ is assumed to be a nonsingular matrix for all q and p, and $J_2(q,p)$ is skew-symmetric. $u \in \mathbb{R}^{n_u}$ with $m \geq n_u$ represents the control input, and the matrix $G(q) \in \mathbb{R}^{m\times n_u}$ is assumed to be a full-rank matrix for all q. $w(t) \in \mathbb{R}^r$ denotes a standard Wiener process defined on a probability space $(\Omega, \mathcal{F}, \mathcal{P})$, where Ω is a sample space, $\mathcal{F}$ is the sigma algebra of the observable random events and $\mathcal{P}$ is a probability measure on Ω. A filtration $\mathcal{F}_t$ represents the sigma algebra generated by $\{x(s) \mid 0 \leq s \leq t\}$, where $x := (q^\top, p^\top)^\top \in \mathbb{R}^n$ with $n = 2m$. $\Xi(q,p) \in \mathbb{R}^{m\times r}$ represents the noise port.

Remark 1: The system of the form (1) with $m = n_u$, $J_1 = I$ and $J_2 = O$ represents a full-actuated typical mechanical system, where I and O denote the identity and zero matrices, respectively. The system with $m > n_u$ represents an underactuated mechanical one (see, e.g. Ortega et al. (2002b)).

In the sequel, we define the norm of a matrix A as $\|A\| := \sqrt{\lambda_{\max}(A^\top A)}$, where $\lambda_{\max}(\cdot)$ represents the maximum eigenvalue of the argument $(\cdot)$. We suppose that the Hamiltonian H is sufficiently differentiable, and that the input u is an $\mathbb{R}^{n_u}$-valued measurable function and satisfies $E[\int_0^t \|u(s)\|^2 \,\mathrm{d}s] < \infty$ with the expectation with respect to the measure $\mathcal{P}$ denoted by $E[\cdot]$. It is also supposed that $\Xi(q,p)$ satisfies the local Lipschitz condition and the linear growth condition, i.e for all q and p, there exists a positive constant K_Ξ such that

$$\|\Xi(q,p)\|^2 \leq K_\Xi(1 + \|x\|^2). \tag{2}$$

We define the following region for any $\delta_0, \delta_1 \in \mathbb{R},\ 0 < \delta_0 < \delta_1$:

$$\begin{aligned} \mathcal{D}(\delta_0,\delta_1) &:= \{x = (q^\top, p^\top)^\top \in \mathbb{R}^n \big| \ \delta_0 < \|x\| < \delta_1\} \\ \bar{\mathcal{D}}(\delta_0,\delta_1) &:= \{x = (q^\top, p^\top)^\top \in \mathbb{R}^n \big| \ \delta_0 \leq \|x\| \leq \delta_1\}. \end{aligned} \tag{3}$$

In order to calculate the expected time variation of a stochastic Lyapunov function, defined later, along the sample path x governed by Eq. (1), we define the infinitesimal generator.

Definition 2.1: Consider the following Itô-type stochastic system:

$$\mathrm{d}x = f(x)\,\mathrm{d}t + g(x)u\,\mathrm{d}t + h(x)\,\mathrm{d}w, \tag{4}$$

where $f(x) \in \mathbb{R}^n$, $g(x) \in \mathbb{R}^{n\times n_u}$ and $h(x) \in \mathbb{R}^{n\times r}$ are sufficiently differentiable functions. Then, the infinitesimal generator for the stochastic process of the system (4) is defined as

$$\mathcal{L}_u(\cdot) := \frac{\partial(\cdot)}{\partial x}(f + gu) + \frac{1}{2}\mathrm{tr}\left\{\frac{\partial^2(\cdot)}{\partial x^2}hh^\top\right\}, \tag{5}$$

where $\mathrm{tr}\{\cdot\}$ represents the trace of the argument. We can obtain the expectation of the time variation of a stochastic Lyapunov function $V(x)$ by calculating $\mathcal{L}_u(V)$ along a sample path x with an input u.

Then, we introduce the notion of (Q_0, Q_1, ρ)-stability due to Kushner (1967) in order to consider the stochastic bounded stability.

Definition 2.2: (Kushner (1967)) The systems is (Q_0, Q_1, ρ)-stable if and only if for any initial condition $x(0) \in Q_0 \subset \mathbb{R}^n$, the probability with respect to a sample path $x(t)$ satisfies

$$\mathcal{P}\{x(t) \in Q_1 \subset \mathbb{R}^n, \quad \text{for } 0 \leq t < \infty\} \geq \rho.$$

3 Main results

We consider the following stochastic Lyapunov function $V(x)$ and the feedback input:

$$V(x) = \frac{a_1}{2} p^\top M(q)^{-1} p + a_2 \frac{\partial U(q)}{\partial q} M(q)^{-1} p + a_1 U(q) \tag{6}$$

$$u = -C(q,p)y, \tag{7}$$

where a_1 and a_2 are positive constants, and a positive-definite matrix $C(q,p) \in \mathbb{R}^{n_u \times n_u}$ represents the feedback gain. They are design parameters, and should be chosen later. Particularly, a_1 and a_2 should be chosen so that $V(x)$ becomes positive definite. The reasons why we use the control input as the feedback of the output y defined in Eq. (1), and why the stochastic Lyapunov function $V(x)$ is defined differently from an energy-based Lyapunov function which is often used in conventional passivity-based control are as follows. In terms of systems modeling, the input and the output of SPHSs as well as deterministic Hamiltonian systems are defined so that they are the effort and the flow variables, whose product is always power. For example, in the case of a mechanical system, the input/output pair defined in Eq. (1) represents the generalized force and velocity. So, the negative feedback of the output such as Eq. (7) efficiently decreases the system energy, i.e. the Hamiltonian $H(q,p)$. However, the time variation of the energy-based Lyapunov function depends only on a part of the state. Meanwhile, since the time variation of the stochastic Lyapunov function defined in Eq. (6) involves all of the state variables, the boundedness of the state can be analyzed in the proposed method.

The closed-loop system of (1) with the feedback controller (7) is given by

$$\begin{pmatrix} \mathrm{d}q \\ \mathrm{d}p \end{pmatrix} = \begin{pmatrix} 0 & J_1(q,p) \\ -J_1(q,p)^\top & J_2(q,p) - \bar{D}(q,p) \end{pmatrix} \begin{pmatrix} \frac{\partial H(q,p)}{\partial q}^\top \\ \frac{\partial H(q,p)}{\partial p}^\top \end{pmatrix} \mathrm{d}t + \begin{pmatrix} 0 \\ \Xi(q,p) \end{pmatrix} \mathrm{d}w, \tag{8}$$

with the new dissipation matrix

$$\bar{D}(q,p) = D(q,p) + G(q)C(q,p)G(q)^\top. \tag{9}$$

Now, we calculate the expected time variation of the Lyapunov function $V(x)$ along the closed-loop system (8) under the input u in (7). From Eqs. (5) and (6), and that $J_2(q,p)$ is skew-symmetric, $\mathcal{L}_u(V)$ is obtained as

$$\begin{aligned}
\mathcal{L}_u(V) &= \frac{\partial V}{\partial q} J_1 \frac{\partial H}{\partial p}^\top + \frac{\partial V}{\partial p} \left(-J_1^\top \frac{\partial H}{\partial q}^\top + (J_2 - \bar{D}) \frac{\partial H}{\partial p}^\top \right) + \frac{1}{2} \mathrm{tr} \left\{ \frac{\partial^2 V}{\partial p^2} \Xi \Xi^\top \right\} \\
&= \left(\frac{a_1}{2} p^\top \frac{\partial M^{-1} p}{\partial q} + a_2 \frac{\partial U}{\partial q} \frac{\partial M^{-1} p}{\partial q} + a_2 p^\top M^{-1} \frac{\partial^2 U}{\partial q^2} + a_1 \frac{\partial U}{\partial q} \right) J_1 M^{-1} p \\
&\quad + \left(a_1 p^\top M^{-1} + a_2 \frac{\partial U}{\partial q} M^{-1} \right) \left(-J_1^\top \left(\frac{1}{2} \frac{\partial M^{-1} p}{\partial q}^\top p + \frac{\partial U}{\partial q}^\top \right) + (J_2 - \bar{D}) M^{-1} p \right) \\
&\quad + \frac{1}{2} \mathrm{tr} \left\{ \frac{\partial^2 V}{\partial p^2} \Xi \Xi^\top \right\}
\end{aligned}$$

$$= -a_2\frac{\partial U}{\partial q}M^{-1}J_1^\top\frac{\partial U}{\partial q}^\top - p^\top M^{-1}\left(a_1\bar{D} - a_2\frac{\partial^2 U}{\partial q^2}J_1\right)M^{-1}p - 2\frac{\partial U}{\partial q}\Sigma_{12}p$$
$$+\frac{1}{2}\operatorname{tr}\left\{\frac{\partial^2 V}{\partial p^2}\Xi\Xi^\top\right\}, \tag{10}$$

where

$$\Sigma_{12} := \frac{a_2}{4}M^{-1}J_1^\top\frac{\partial M^{-1}p}{\partial q}^\top - \frac{a_2}{2}\frac{\partial M^{-1}p}{\partial q}J_1M^{-1} - \frac{a_2}{2}M^{-1}(J_2 - \bar{D})M^{-1}.$$

We can define symmetric matrices Σ_{11} and Σ_{22} such that

$$a_2\frac{\partial U}{\partial q}M^{-1}J_1^\top\frac{\partial U}{\partial q}^\top = \frac{\partial U}{\partial q}\Sigma_{11}\frac{\partial U}{\partial q}^\top,$$
$$p^\top M^{-1}\left(a_1\bar{D} - a_2\frac{\partial^2 U}{\partial q^2}J_1\right)M^{-1}p = p^\top\Sigma_{22}p$$

holds for all q and p, as

$$\Sigma_{11} := \left(a_2M^{-1}J_1^\top + a_2J_1M^{-1}\right)/2, \tag{11}$$
$$\Sigma_{22} := \left(M^{-1}\left(a_1\bar{D} - a_2\frac{\partial^2 U}{\partial q^2}J_1\right)M^{-1} + M^{-1}\left(a_1\bar{D}^\top - a_2J_1^\top\frac{\partial^2 U}{\partial q^2}\right)M^{-1}\right)\Big/2.$$

Using Eq. (11), Eq. (10) is rewritten as

$$\mathcal{L}_u(V) = -\left(\frac{\partial U}{\partial q}, p^\top\right)\begin{pmatrix}\Sigma_{11} & \Sigma_{12}\\ \Sigma_{12}^\top & \Sigma_{22}\end{pmatrix}\begin{pmatrix}\frac{\partial U}{\partial q}^\top\\ p\end{pmatrix} + \frac{1}{2}\operatorname{tr}\left\{\frac{\partial^2 V}{\partial p^2}\Xi\Xi^\top\right\}$$
$$=: -\left(\frac{\partial U}{\partial q}, p^\top\right)\Sigma(x, a_1, a_2, C)\begin{pmatrix}\frac{\partial U}{\partial q}^\top\\ p\end{pmatrix} + \frac{1}{2}\operatorname{tr}\left\{\frac{\partial^2 V}{\partial p^2}\Xi\Xi^\top\right\}, \tag{12}$$

where the matrix $\Sigma(x, a_1, a_2, C) \in \mathbb{R}^{n\times n}$ becomes symmetric from Eq. (11).

Here, we show the main results on bounded stability of the system (1). After we introduce some basic notations, we provide a lemma to be used for the main theorem.

Definition 3.1: (Khalil (1996)) A continuous function $\alpha : [0,\infty) \mapsto [0,\infty)$ is said to belong to class $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$. It is said to belong to class $\mathcal{K}_\infty$ if $\alpha(s) \to \infty$ as $s \to \infty$.

Lemma 3.2: *Consider the system of the form* (1)*, the feedback input u in* (7)*, and a given region $\mathcal{D}(\delta_0, \delta_1)$ in* (3) *with some δ_0 and δ_1. Suppose that there exist a positive constant M_m and a positive definite function γ_m satisfying the following inequalities for all $x \in \{x \in \mathbb{R}^n \mid \|x\| < \delta_1\}$:*

$$M_m \le \|M(q)\|, \tag{13}$$
$$\gamma_m(\|x\|) \le \left\|\begin{pmatrix}\frac{\partial U(q)}{\partial q}^\top\\ p\end{pmatrix}\right\|. \tag{14}$$

Then, a sufficient condition under which $\mathcal{L}_u(V)$ with respect to V defined in (6) *becomes strictly negative in the region $\mathcal{D}(\delta_0, \delta_1)$ is that there exist positive constants a_1 and a_2 and the gain matrix $C(q,p)$ in* (7) *(see also Eqs.* (9) *and* (11)*) such that* *(i) there exist class $\mathcal{K}_\infty$ functions α_m, α_M and a positive constant $K_{\Sigma m}(a_1, a_2, C)$ satisfying*

$$\alpha_m(\|x\|) \leq V(x) \leq \alpha_M(\|x\|) \tag{15}$$

$$K_{\Sigma m}(a_1, a_2, C) \leq \|\Sigma(x, a_1, a_2, C)\|, \qquad \forall x \in \{x \in \mathbb{R}^n \mid \|x\| < \delta_1\}; \tag{16}$$

(ii) the following inequality holds:

$$\gamma_m(\delta_0) \geq \sqrt{\frac{a_1 r K_\Xi(1+\delta_1^2)}{2M_m K_{\Sigma m}(a_1, a_2, C)}}.$$

Proof Under the condition (i), we evaluate $\mathcal{L}_u(V)$ in (12). From the linear growth condition (2), and the boundedness of $M(q)$ in Eq. (13), the second term in Eq. (12), which results from the noise effect in Itô calculus, is evaluated as

$$\begin{aligned}\frac{1}{2}\operatorname{tr}\left\{\frac{\partial^2 V}{\partial p^2}\Xi\Xi^\top\right\} &= \frac{a_1}{2}\sum_{i=1}^{r}\lambda_i\left(\Xi^\top M^{-1}\Xi\right)\\ &\leq \frac{a_1 r}{2}\lambda_{\max}\left(\Xi^\top M^{-1}\Xi\right)\\ &\leq \frac{a_1 r}{2M_m}K_\Xi(1+\|x\|^2).\end{aligned} \tag{17}$$

From Eqs. (14), (15), (16) and (17), $\mathcal{L}_u(V)$ is evaluated in $\mathcal{D}(\delta_0, \delta_1)$ as

$$\begin{aligned}\mathcal{L}_u(V) &\leq -K_{\Sigma m}\left\|\begin{pmatrix}\frac{\partial U}{\partial q}^\top\\ p\end{pmatrix}\right\|^2 + \frac{a_1 r}{2M_m}K_\Xi(1+\|x\|^2)\\ &\leq -K_{\Sigma m}\gamma_m(\|x\|)^2 + \frac{a_1 r}{2M_m}K_\Xi(1+\|x\|^2)\\ &< -K_{\Sigma m}\gamma_m(\delta_0)^2 + \frac{a_1 r}{2M_m}K_\Xi(1+\delta_1^2).\end{aligned} \tag{18}$$

From Eq. (18), the condition (ii) implies that $\mathcal{L}_u(V)$ becomes strictly negative in $\mathcal{D}(\delta_0, \delta_1)$. □

Now, we show the main theorem.

Theorem 3.3: *Consider the system of the form (1) and the feedback input u in (7). For any bounded region parameter $\delta_1 \in \mathbb{R}, \delta_1 > 0$ and any assigned probability $\rho \in \mathbb{R},\ 0 < \rho < 1$, the least upper bound for the initial region parameter $\bar{\delta}_0$ is assigned by*

$$\bar{\delta}_0 = \alpha_M^{-1}\left((1-\rho)\alpha_m(\delta_1)\right). \tag{19}$$

Then, under the conditions in Lemma 3.2 with the bounded region parameter δ_1 and any initial region parameter δ_0 such that $0 < \delta_0 < \bar{\delta}_0$, the system is (Q_0, Q_1, ρ)-stable (see Definition 2.2), where Q_0 and Q_1 are given by

$$\begin{aligned}Q_0 &= \{x \in \mathbb{R}^n \mid x \in \bar{\mathcal{D}}(\delta_0, \bar{\delta}_0)\}\\ Q_1 &= \{x \in \mathbb{R}^n \mid \|x\| < \delta_1\}.\end{aligned} \tag{20}$$

Further, the following probability inequality is achieved:

$$\mathcal{P}\left\{\sup_{0\le t<\infty}\|x(t)\|<\delta_1\right\}>\rho. \tag{21}$$

Before proving the theorem, the stopped process (Kushner (1967)) is introduced.

Definition 3.4: (Kushner (1967)) Define $t\cap s := \min\{t,s\}$. Suppose that $\tau_{\mathcal{D}}$ is the first time of exit of the process $x(s)$ from an open set $\mathcal{D}$, i.e., $\tau_{\mathcal{D}} := \inf\{t \ge 0 \mid x(t) \notin \mathcal{D}\}$. Then, the stopped process $x(t\cap\tau_{\mathcal{D}})$ is defined as

$$x(t\cap\tau_{\mathcal{D}}) := \begin{cases} x(t) & t<\tau_{\mathcal{D}} \\ x(\tau_{\mathcal{D}}) & t\ge\tau_{\mathcal{D}} \end{cases}.$$

Here, we prove Theorem 3.3.

Proof For the proof, we define the following region with respect to the stochastic Lyapunov function:

$$\mathcal{D}^V(\lambda_0,\lambda_1) := \{x\in\mathbb{R}^n \mid \lambda_0 < V(x) < \lambda_1\}. \tag{22}$$

From the assumption of the theorem and Lemma 3.2, $\mathcal{L}_u(V)<0$ in $\mathcal{D}(\delta_0,\delta_1)$ holds. Then, it follows from Eqs. (15) and (22) that $\mathcal{L}_u(V)<0$ in $\mathcal{D}^V(\alpha_M(\delta_0),\alpha_m(\delta_1))$ holds (Figure 1 may help to describe them). In what follows, we simply write $\mathcal{D}^V$ as the region $\mathcal{D}^V(\alpha_M(\delta_0),\alpha_m(\delta_1))$. Then, from Dynkin's formula (Dynkin (1965), Øksendal (1998)), for $0\le s\le t$, we have

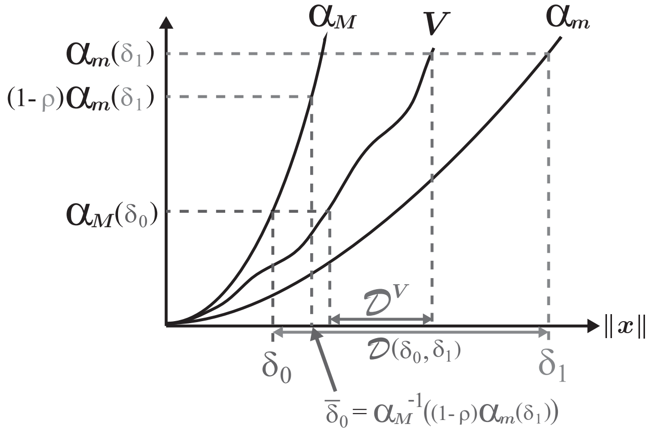

Figure 1. Illustration of the regions $\mathcal{D}(\delta_0,\delta_1)$ and $\mathcal{D}^V(\alpha_M(\delta_0),\alpha_m(\delta_1))$

$$E[V(x(t\cap\tau_{\mathcal{D}^V}))]-E[V(x(s))] = E\left[\int_s^{t\cap\tau_{\mathcal{D}^V}} \mathcal{L}_u(V(x(\bar{t})))\,\mathrm{d}\bar{t}\right] < 0. \tag{23}$$

Since $E[V(x(t\cap\tau_{\mathcal{D}^V}))|\mathcal{F}_s]<V(x(s))$ holds from Eq. (23), $\{V(x(t\cap\tau_{\mathcal{D}^V}));\ t\ge 0\}$ is a nonnegative supermartingale. It follows that, for any $t\ge 0$

$$E[V(x(t\cap\tau_{\mathcal{D}^V}))]<V(x(0)). \tag{24}$$

Since $V(x(t)) = \alpha_m(\delta_1)$ for some t implies that the state x reaches the boundary of the region $\mathcal{D}^V$ (see also Figure 1), we have

$$\mathcal{P}\left\{\sup_{0\le t\le \bar{t}} V(x(t)) \ge \alpha_m(\delta_1)\right\} = \mathcal{P}\left\{V(x(\bar{t}\cap\tau_{\mathcal{D}^V})) = \alpha_m(\delta_1)\right\}\ , \quad \forall \bar{t}\ge 0. \tag{25}$$

Since $V(x)$ is positive definite, the following inequality holds due to Chebyshev's inequality:

$$E\left[V(x(\bar{t}\cap\tau_{\mathcal{D}^V}))\right] \ge \alpha_m(\delta_1)\mathcal{P}\left\{V(x(\bar{t}\cap\tau_{\mathcal{D}^V})) = \alpha_m(\delta_1)\right\}\ , \quad \forall \bar{t}\ge 0. \tag{26}$$

From Eqs. (24), (25), (26), and by taking a limit as $\bar{t}\to\infty$, we obtain

$$\frac{V(x(0))}{\alpha_m(\delta_1)} > \mathcal{P}\left\{\sup_{0\le t<\infty} V(x(t)) \ge \alpha_m(\delta_1)\right\}. \tag{27}$$

It follows from Eq. (15) that if $\|x(0)\| \le \bar{\delta}_0$, then $V(x(0)) \le (1-\rho)\alpha_m(\delta_1)$ holds (see also Figure 1). Therefore, if $x(0)$ is chosen from Q_0 in (20), we have

$$\mathcal{P}\left\{\sup_{0\le t<\infty} V(x(t)) < \alpha_m(\delta_1)\right\} > 1 - \frac{V(x(0))}{\alpha_m(\delta_1)} \ge 1 - \frac{(1-\rho)\alpha_m(\delta_1)}{\alpha_m(\delta_1)} = \rho. \tag{28}$$

Since $V(x(t)) < \alpha_m(\delta_1)$ is a sufficient condition for $\|x(t)\| < \delta_1$ from Eq. (15) (see also Figure 1), Eq. (28) implies that the asserted probability inequality holds. □

Finally, we summarize a design procedure in the proposed method:

- Step 1 : Set any bounded region parameter $\delta_1 \in \mathbb{R}, \delta_1 > 0$ and any assigned probability $\rho \in \mathbb{R},\ 0 < \rho < 1$.
- Step 2 : Choose positive constants a_1 and a_2 such that $V(x)$ in (6) becomes positive definite. Calculate a positive constant K_Ξ and choose class $\mathcal{K}_\infty$ functions α_m and α_M satisfying Eqs. (2) and (15). Calculate a positive constant M_m and choose a positive-definite function γ_m satisfying Eqs. (13) and (14) in the region $\{x \in \mathbb{R}^n \mid \|x\| < \delta_1\}$.
- Step 3 : Calculate the least upper bound for the initial region parameter $\bar{\delta}_0$ from Eq. (19), and set any initial region parameter δ_0 such that $0 < \delta_0 < \bar{\delta}_0$.
- Step 4 : Choose a positive-definite matrix $C(q,p)$ such that Eq. (16) and the condition (ii) in Lemma 3.2 hold.

  Unless the above $C(q,p)$ is found, start again from Step 2 with another pair of a_1 and a_2, and retry the rest procedures.

Eventually, the closed loop system (1) with the feedback input u in (7) becomes (Q_0, Q_1, ρ)-stable, where the initial region Q_0 and bounded region Q_1 are given by Eq. (20) with δ_0 and δ_1.

From the definition in Eq. (6), $V(x)$ becomes positive definite, by letting a_1 sufficiently larger than a_2. Although the upper or lower bounds M_m, γ_m, α_m and α_M can sometimes be analytically obtained, otherwise, they should be numerically calculated. Since it is difficult to analytically calculate $K_{\Sigma m}$ and solve the condition (ii) in many cases, they should be solved numerically. Although we have provided sufficient conditions for the deign parameters a_1, a_2 and $C(q,p)$, we have not obtained a systematic solution method to find them yet.

4 Numerical examples

This section exhibits applications of the proposed bounded stabilization method. First, in Subsection 4.1, we consider the stabilization of a two-link robot manipulator in the presence of noise,

Table 1. Physical parameters

m_i	Mass of the ith link	[kg]
l_i	Length of the ith link	[m]
l_{ci}	Length to the center of gravity	[m]
I_i	Inertia of the ith link	[kg m^2]
d_i	Viscous friction coefficient of the ith link	[Nms/rad]
g	Gravity acceleration	[m/s^2]

whose dynamics is described as the first case in Remark **1**. Second, in Subsection 4.2, we consider the stabilization of an inertia wheel pendulum in the presence of noise, which corresponds to the second case in Remark **1**.

4.1 *Stabilization of the robot manipulator*

Let us consider a two-link robot manipulator moving on a vertical plane depicted in Figure 2. As in the figure, the joint angles of the first and the second links are denoted by θ_1 and θ_2, and the control torques are denoted by u_1 and u_2, respectively. The physical parameters of this apparatus are summarized in Table 1. The dynamics of this apparatus is described by a full-

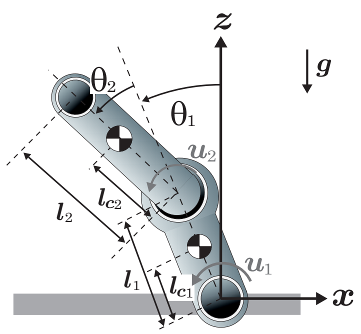

Figure 2. Two-link robot manipulator

actuated mechanical system of the form (1), where $q := (\theta_1, \theta_2)^\top \in \mathbb{R}^2$ and $p = M(q)\dot{q} \in \mathbb{R}^2$ with the inertia matrix

$$M(q) = \begin{pmatrix} I_1 + I_2 + m_1 l_{c1}^2 + m_2(l_1^2 + l_{c2}^2 + 2l_1 l_{c2}\cos q_2) & I_2 + m_2(l_{c2}^2 + l_1 l_{c2}\cos q_2) \\ I_2 + m_2(l_{c2}^2 + l_1 l_{c2}\cos q_2) & I_2 + m_2 l_{c2}^2 \end{pmatrix},$$

$G = I$, and the structure and dissipation matrices $J_1 = I$, $J_2 = O$ and $D = \mathrm{diag}\{d_1, d_2\}$, respectively, where $\mathrm{diag}\{\cdot\}$ denotes a (block) diagonal matrix. The state and the Hamiltonian are given by $x = (q^\top, p^\top)^\top \in \mathbb{R}^4$ and

$$H_0(q, p) = \frac{1}{2} p^\top M(q)^{-1} p + U_0(q),$$

with the potential energy

$$U_0(q) = ((m_1 l_{c1} + m_2 l_1)\cos q_1 + m_2 l_{c2}\cos(q_1 + q_2))g.$$

Then, in order to appropriately shape the potential energy of the system, we assign the following pre-feedback:

$$u = -K_q q + \frac{\partial U_0(q)}{\partial q}^\top + \bar{u}, \tag{29}$$

where a symmetric positive definite matrix K_q represents a gain matrix, and $\bar{u} \in \mathbb{R}^2$ represents a new control input. The pre-feedback (29) is designed by the stochastic generalized canonical transformation (Theorem 2 in Satoh and Fujimoto (2013)), so that the closed-loop system preserves the stochastic port-Hamiltonian structure of the form (1). For design methods preserving the deterministic Hamiltonian structure, see, e.g. van der Schaft (1996), Fujimoto and Sugie (2001), Ortega et al. (2002a). Under the pre-feedback (29), the dynamics of the closed-loop system is again described by an SPHS of the form (1) with the new Hamiltonian

$$H(q,p) = \frac{1}{2} p^\top M(q)^{-1} p + U(q),$$
$$U(q) = \frac{1}{2} q^\top K_q q,$$

and with new control input $\bar{u}$. The output y is given by $y = M(q)^{-1}p = \dot{q}$. Since we use the feedback input of the form (7) for $\bar{u}$, the controller is consequently given by

$$u = -K_q q + \frac{\partial U_0(q)}{\partial q}^\top - C(q,p)M(q)^{-1}p. \tag{30}$$

The concrete parameters used in the simulation are $m_1 = m_2 = 1$ [kg], $l_1 = 1, l_{c1} = 0.5, l_2 = 2, l_{c2} = 1$ [m], $I_1 = 8.3 \times 10^{-2}, I_2 = 3.3 \times 10^{-1}$ [kg m^2] and $d_1 = d_2 = 0.5$ [Nms/rad]. We consider the noise port as $\Xi = \text{diag}\{h_{10} + h_{11}q_1 + h_{12}\dot{q}_1, h_{20} + h_{21}q_2 + h_{22}\dot{q}_2\}$ with $h_{ij} = 1, i \in \{1,2\}, j \in \{0,1,2\}$. We set design parameters as $K_q = \text{diag}\{k_1, k_2\}$ and $C(q,p) = \text{diag}\{c_1, c_2\}$, respectively. Then, γ_m in Eq. (14) is given by $\gamma_m = \min\{1, k_1, k_2\}\|x\|$. As Step 1 in the summary of the proposed method, we choose the bounded region parameter as $\delta_1 = 20$ and the assigned probability as $\rho = 0.85$. As Step 2, we numerically calculate K_Ξ in Eq. (2) and M_m in Eq. (13) in the range of $-\pi \le q_1, q_2 \le \pi$ and $-5 \le \dot{q}_1, \dot{q}_2 \le 5$. The results are $K_\Xi = 3$ and $M_m = 1.5$, respectively. The stochastic Lyapunov function in Eq. (6) is rewritten as

$$V(x) = \frac{1}{2} x^\top P(x) x \ , \quad P(x) := \begin{pmatrix} a_1 K_q & a_2 K_q M(q)^{-1} \\ a_2 M(q)^{-1} K_q & a_1 M(q)^{-1} \end{pmatrix}.$$

The necessary and sufficient condition for $V(x)$ to be positive definite is that $a_1^2 M(q) - a_2^2 K_q$ becomes positive definite, which is given by the Schur complement, and that K_q is positive definite and $M(q)$ is nonsingular. Here, we empirically choose the design parameters as $a_1 = 15$, $a_2 = 1$ and $k_1 = k_2 = 20$, so that $a_1^2 M(q) - a_2^2 K_q$ becomes positive definite and Eq. (15) in the condition (i) of Lemma 3.2 holds. We numerically have $\alpha_m = 304\|x\|^2$ and $\alpha_M = 385\|x\|^2$. As Step 3, from those parameters, the least upper bound for the initial region parameter $\bar{\delta}_0$ in Eq. (19) is calculated as $\bar{\delta}_0 = 12.6$. Then, we choose the initial region parameter as $\delta_0 = 4.4$ so that $0 < \delta_0 < \bar{\delta}_0$ holds. As Step 4, we empirically choose the rest design parameters $c_1 = c_2 = 20$ so that Eq. (16) holds with $K_{\Sigma m} = 650$ and the condition (ii) holds, respectively. Consequently, Theorem 3.3 guarantees that the system is (Q_0, Q_1, ρ)-stable with $Q_0 = \{x \in \mathbb{R}^4 \mid 4.4 \le \|x\| \le 12.6\}$ and $Q_1 = \{x \in \mathbb{R}^4 \mid \|x\| < 20\}$, and that $\mathcal{P}\left\{\sup_{0 \le t < \infty} \|x(t)\| < 20\right\} > \rho = 0.85$ holds.

We set the initial state as $q_1(0) = -80\pi/180, q_2(0) = 40\pi/180$ [rad] and $\dot{q}_1 = 0.7, \dot{q}_2 = 0.5$

[rad/s], which implies

$$x(0) = (q(0)^\top, p(0)^\top)^\top = (-1.3963, 0.6981, 3.9846, 2.1322)^\top.$$

Since the norm of the above initial state is calculated as $\|x(0)\| = 4.7812$, $x(0) \in Q_0$ holds. The simulation is executed on $t \in [0, 30]$ [s]. The simulation results are shown in Figures 3-7. Figure 3 denotes the responses of the joint angles q_1 and q_2. Figure 4 denotes those of the momentum p_1 and p_2. Since it might be easier to understand the motion of the manipulator with the angular velocity than the momentum, the responses of $\dot{q}_1$ and $\dot{q}_2$ are shown in Figure 5. Those figures show that the state is approaching to the origin and fluctuates due to the persistent disturbances. Figure 6 denotes the time history of the stochastic Lyapunov function $V(x)$ in Eq. (6) along the closed-loop system with the feedback input u in (30). This figure shows that the Lyapunov function decreases to zero and fluctuates around zero. Finally, Figure 7 denotes the time history of the norm of the state $\|x(t)\|$ and the bounded region parameter δ_1, where Theorem 3.3 guarantees that $\|x(t)\|$ is below $\delta_1 = 20$ for all $t \geq 0$ with probability more than $\rho = 0.85$. Furthermore, we generate nine other sample paths x's under the same controller, and exhibit the maximum envelop curve of the norm of all sample paths in the same figure in dotted line. Although the simulation is executed on a finite time interval, this figure implies that the designed controller achieves the control objective.

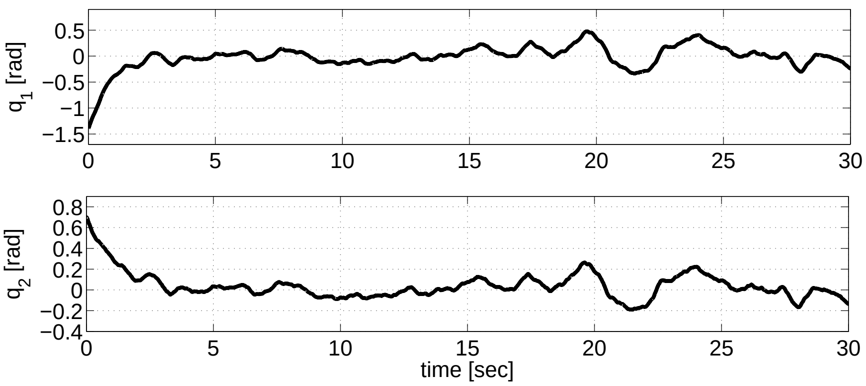

Figure 3. Responses of q_1 and q_2

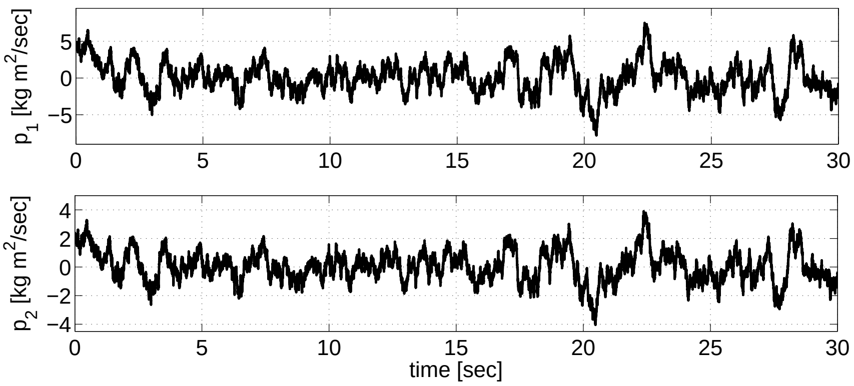

Figure 4. Responses of p_1 and p_2

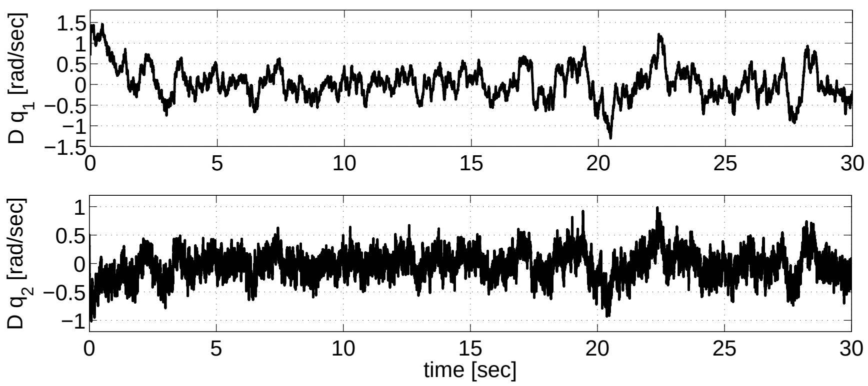

Figure 5. Responses of $\dot{q}_1$ and $\dot{q}_2$

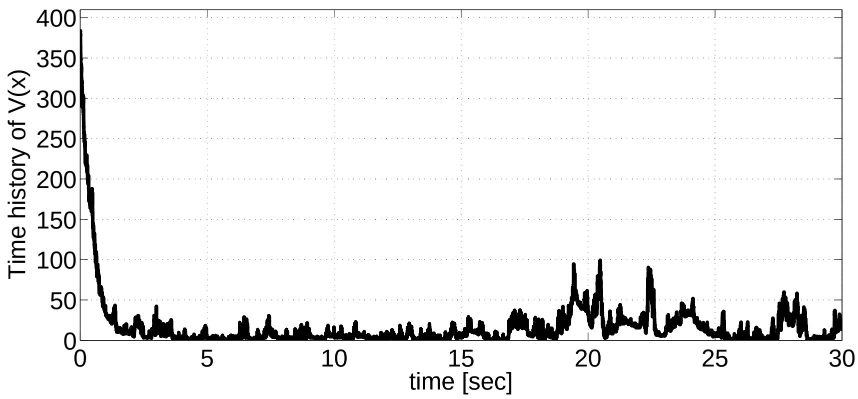

Figure 6. Time history of stochastic Lyapunov function $V(x)$

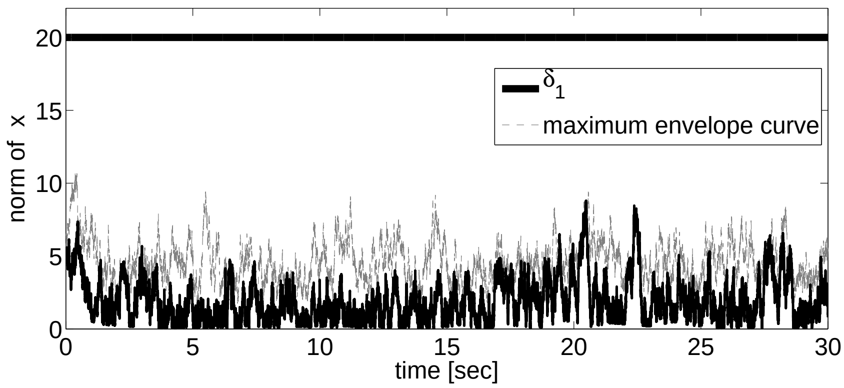

Figure 7. Time history of $\|x\|$ and maximum envelop curve of 10 samples

4.2 *Stabilization of the inertia wheel pendulum*

Here, we consider an inertia wheel pendulum depicted in Figure 8. The angles with respect to the vertical of the pendulum and the wheel are denoted by θ_1 and θ_2, respectively. The control torque acting between the pendulum and wheel is denoted by u. The physical parameters of this apparatus are summarized in Table 2. The dynamics of this apparatus is described by an underactuated mechanical system of the form (1), where $q := (\theta_1, \theta_2)^\top \in \mathbb{R}^2$ and $p = M\dot{q} \in \mathbb{R}^2$ with the inertia matrix $M = \mathrm{diag}\{I_1, I_2\}$, $G = (-1, 1)^\top$, and the structure and dissipation

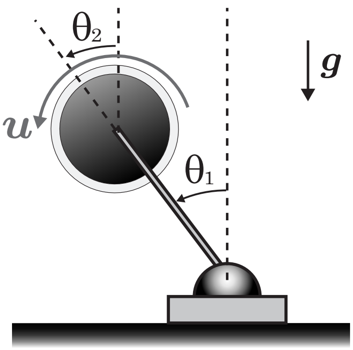

Figure 8. Inertia wheel pendulum

Table 2. Physical parameters

m	Mass of the wheel pendulum	[kg]
l	Length of the pendulum	[m]
I_1	Inertia of the pendulum	[kg m^2]
I_2	Inertia of the wheel	[kg m^2]
g	Gravity acceleration	[m/s^2]

matrices $J_1 = I$, $J_2 = O$ and $D = O$, respectively. The state and the Hamiltonian are given by $x = (q^\top, p^\top)^\top \in \mathbb{R}^4$ and

$$H_0(q, p) = \frac{1}{2} p^\top M^{-1} p + U_0(q),$$

with the potential energy

$$U_0(q) = mgl(\cos q_1 - 1).$$

Based on interconnection and damping assignment passivity-based control (IDA-PBC) method for the deterministic Hamiltonian systems in Ortega et al. (2002b), we assign the following pre-feedback:

$$u = \frac{\bar{m}_2}{\bar{m}_1 + \bar{m}_2} mgl \sin q_1 - k \frac{\bar{m}_1 \bar{m}_3 - \bar{m}_2^2}{I_2(\bar{m}_1 + \bar{m}_2)} (q_2 + k_q q_1) + \bar{u} \ , \quad k_q := -\frac{I_1(\bar{m}_2 + \bar{m}_3)}{I_2(\bar{m}_1 + \bar{m}_2)}, \tag{31}$$

where the parameters $\bar{m}_1, \bar{m}_2$ and $\bar{m}_3$ satisfy the following three conditions: $\bar{m}_1 > 0$, $\bar{m}_1 \bar{m}_3 - \bar{m}_2^2 > 0$ and $\bar{m}_1 + \bar{m}_2 < 0$, and a positive constant k represents a gain. $\bar{u} \in \mathbb{R}$ represents a new control input. Under the pre-feedback (31), the dynamics of the closed-loop system is again described by an SPHS of the form (1) with the new Hamiltonian

$$H(q, p) = \frac{1}{2} p^\top \bar{M}^{-1} p + U(q) \ , \quad \bar{M} := \begin{pmatrix} \bar{m}_1 & \bar{m}_2 \\ \bar{m}_2 & \bar{m}_3 \end{pmatrix}$$

$$U(q) = \frac{I_1 mgl}{\bar{m}_1 + \bar{m}_2} (\cos q_1 - 1) + \frac{k}{2} (q_2 + k_q q_1)^2$$

and the new structure matrices $J_1 = M^{-1} \bar{M}$ and $J_2 = O$. The new control input and the output are given by $\bar{u}$ and $y = G^\top \bar{M}^{-1} p$, respectively. Since we use the feedback input of the form (7) for $\bar{u}$, the controller is consequently given by

$$u = \frac{\bar{m}_2}{\bar{m}_1 + \bar{m}_2} mgl \sin q_1 - k \frac{\bar{m}_1 \bar{m}_3 - \bar{m}_2^2}{I_2(\bar{m}_1 + \bar{m}_2)}(q_2 + k_q q_1) - cG^\top \bar{M}^{-1} p. \tag{32}$$

The concrete parameters used in the simulation are $m = 1$ [kg], $l = 1$ [m] and $I_1 = 0.1, I_2 = 0.2$ [kg m^2]. Then we have M_m in Eq. (13) with respect to the new inertia matrix $\bar{M}$ as $M_m = (7 - 3\sqrt{5})/2$. We consider the noise port as $\Xi = \mathrm{diag}\{h_{10} + h_{11}q_1 + h_{12}\dot{q}_1, h_{20} + h_{21}q_2 + h_{22}\dot{q}_2\}$ with $h_{ij} = 0.5, i \in \{1, 2\}, j \in \{0, 1, 2\}$.

As Step 1 in the summary of the proposed method, we choose the bounded region parameter as $\delta_1 = 10$ and the assigned probability as $\rho = 0.85$. As Step 2, we suppose that there exists a positive constant K_{Um} such that $K_{Um}\|q\|^2 \leq \left\| \frac{\partial U(q)}{\partial q} \right\|^2$, that is, γ_m in Eq. (14) should be given by $\gamma_m = \min\{1, \sqrt{K_{Um}}\}\|x\|$, and we numerically calculate K_Ξ in Eq. (2) and K_{Um} in the range of $-\pi \leq q_1, q_2 \leq \pi$ and $-10 \leq \dot{q}_1, \dot{q}_2 \leq 10$. The results are $K_\Xi = 7.49 \times 10^{-1}$ and $K_{Um} = 3.73 \times 10^{-1}$, respectively. We empirically choose the design parameters as $\bar{m}_1 = 0.2, \bar{m}_2 = -0.3, \bar{m}_3 = 0.5$, $a_1 = 10$, $a_2 = 0.1$ and $k = 30$ so that the stochastic Lyapunov function $V(x)$ becomes positive definite, and Eq. (15) in the condition (i) of Lemma 3.2 holds. We numerically have $\alpha_m = 9.63\|x\|^2$ and $\alpha_M = 3.27 \times 10^2\|x\|^2$ from those parameters. As Step 3, the least upper bound for the initial region parameter $\bar{\delta}_0$ in Eq. (19) is calculated as $\bar{\delta}_0 = 0.66$. Then, we choose $\delta_0 = 0.48$ so that $0 < \delta_0 < \bar{\delta}_0$ holds. As Step 4, we empirically choose the rest parameter $c = 50$ so that Eq. (16) holds with $K_{\Sigma m} = 6.21 \times 10^4$ and the condition (ii) holds, respectively. Consequently, Theorem 3.3 guarantees that the system is (Q_0, Q_1, ρ)-stable with $Q_0 = \{x \in \mathbb{R}^4 \mid 0.48 \leq \|x\| \leq 0.66\}$ and $Q_1 = \{x \in \mathbb{R}^4 \mid \|x\| < 10\}$, and that $\mathcal{P}\left\{\sup_{0 \leq t < \infty} \|x(t)\| < 10\right\} \geq \rho = 0.85$ holds.

We set the initial state as $q_1(0) = -37\pi/180, q_2(0) = 0$ [rad] and $\dot{q}_1 = \dot{q}_2 = 0$ [rad/s], which implies $x(0) = (q(0)^\top, p(0)^\top)^\top = (-0.6458, 0, 0, 0)^\top$. Since the norm of the above initial state is calculated as $\|x(0)\| = 0.6458$, $x(0) \in Q_0$ holds. The simulation is executed on $t \in [0, 50]$ [s]. The simulation results are shown in Figures 9-13. Figure 9 denotes the responses of the angles with respect to the vertical q_1 and q_2. Figure 10 denotes those of the momentum p_1 and p_2. The responses of $\dot{q}_1$ and $\dot{q}_2$ are shown in Figure 11. Those figures show that the state is approaching to the origin and fluctuates due to the persistent disturbances. Figure 12 denotes the time history of the stochastic Lyapunov function $V(x)$ in Eq. (6) along the closed-loop system with the feedback input u in (32). This figure shows that the Lyapunov function decreases to zero and fluctuates around zero. Finally, Figure 13 denotes the time history of the norm of the state $\|x(t)\|$ and the bounded region parameter δ_1, where Theorem 3.3 guarantees that $\|x(t)\|$ is below $\delta_1 = 10$ for all $t \geq 0$ with probability $\rho = 0.85$. Furthermore, we generate nine other sample paths x's under the same controller, and exhibit the maximum envelop curve of the norm of all sample paths in the same figure in dotted line. Although the simulation is executed on a finite time interval, this figure implies that the designed controller achieves the control objective.

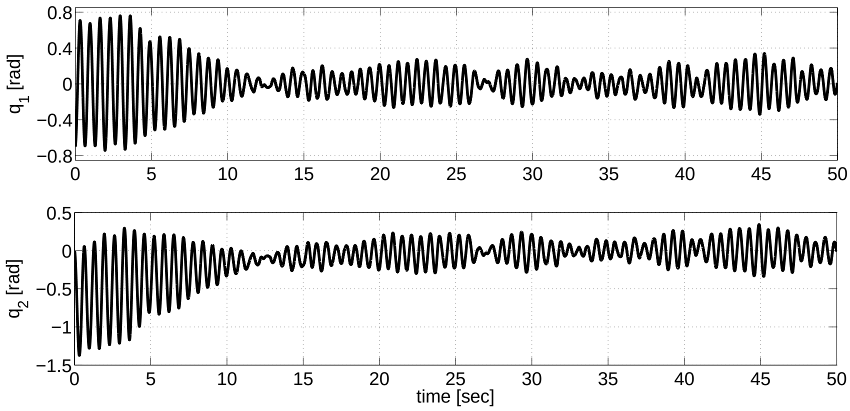

Figure 9. Responses of q_1 and q_2

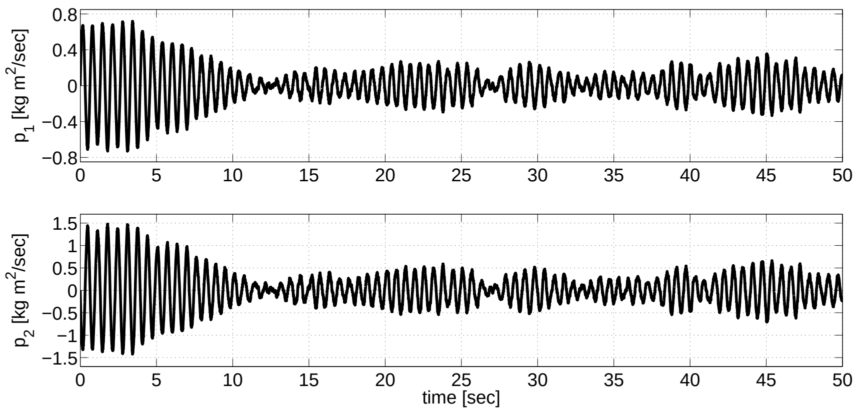

Figure 10. Responses of p_1 and p_2

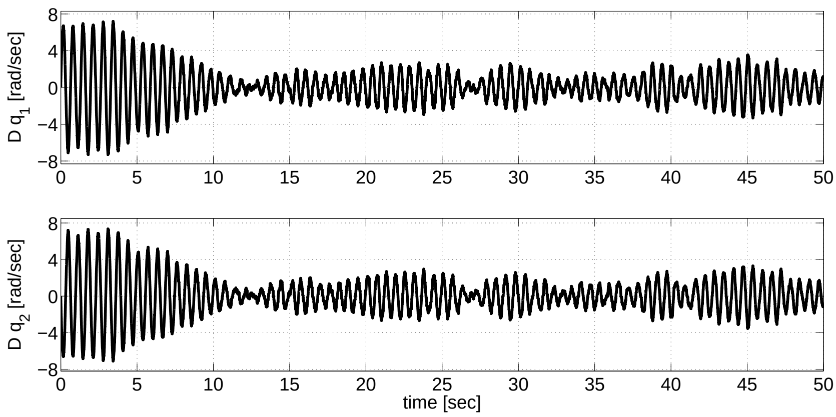

Figure 11. Responses of $\dot{q}_1$ and $\dot{q}_2$

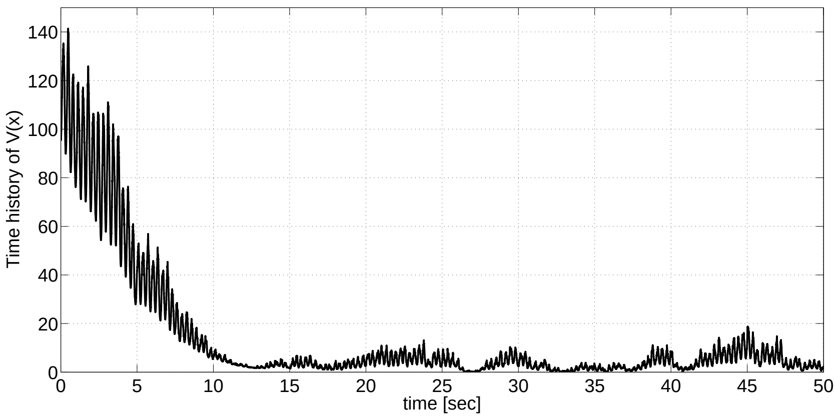

Figure 12. Time history of stochastic Lyapunov function $V(x)$

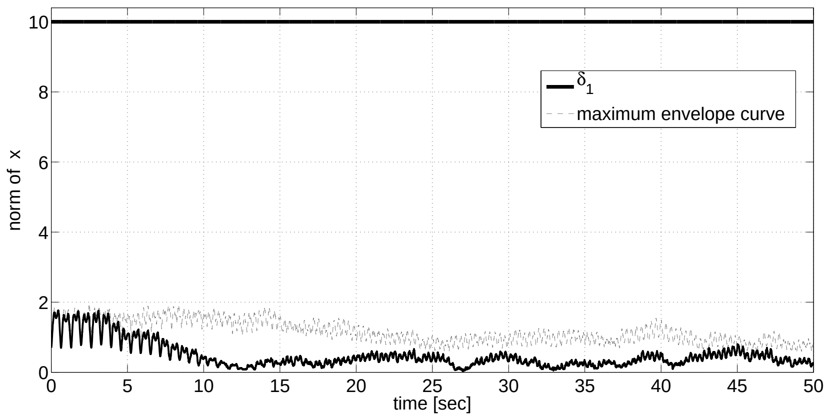

Figure 13. Time history of $\|x\|$ and maximum envelop curve of 10 samples

5 Conclusion

This paper has proposed a stochastic bounded stabilization method for a class of stochastic port-Hamiltonian systems. This includes not only full-actuated, but also underactuated mechanical systems in the presence of noise. We have derived conditions for the controller gain and design parameters under which the state remains bounded in probability for a given probability and bound of the state. Although many conventional stabilization methods assume that the noise vanishes at the origin, the proposed method is applicable to a broader class of mechanical systems under persistent disturbances. The main advantages of the proposed method are that both a bounded region for the state and its achieving probability can be assignable, and that we give not only a stability condition, but also a construction method of a stochastic Lyapunov function.

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