Observer based stochastic trajectory tracking control of mechanical systems

Satoshi Satoh[1] and Kenji Fujimoto[2]

[1]Nagoya University, Furo-cho, Chikusa-ku, Nagoya-shi,
Aichi, 464-8603, Japan; (E-mail: s.satoh@nagoya-u.jp)
[2]Nagoya University, Furo-cho, Chikusa-ku, Nagoya-shi,
Aichi, 464-8603, Japan; (E-mail: fujimoto@nagoya-u.jp)

Abstract: This paper concerns an observer based stochastic trajectory tracking control of mechanical systems. We consider mechanical systems in the presence of noise as stochastic systems and derive a condition for a stabilizing and tracking controller to achieve each control objective in probability. Here we assume that only position measurements can be available. Velocity information is reconstructed by the position information. The condition for the combined controller-observer is derived. Since our method is based on bounded stability, the norm of tracking error with respect to a given desired trajectory remains bounded in probability and the margin of error is assignable.

Keywords: Stochastic stability, Nonlinear stochastic control, Passive stochastic systems, Nonlinear observer, Output feedback control

1. INTRODUCTION

There exist various disturbances such as measurement noise, modeling error and so on in controlling real plants. Since they sometimes cause performance degradation or destabilization of the plant, it is important to consider them. Stochastic control theory is one of the efficient ways which can take such disturbances into account. Some useful results on the deterministic control theory are extended to the stochastic case, e.g. [1, 2].

The authors introduced stochastic port-Hamiltonian systems [3] as extension of port-Hamiltonian systems [4] which can represent not only physical systems but also electrical ones, nonholonomic ones and so on. Stochastic passivity [2] based stabilization framework was also proposed in [3]. Then stochastic trajectory tracking control was considered in the authors' previous work [5]. In [5] bounded stability is achieved, where it is guaranteed that a norm of tracking error becomes arbitrarily small in probability. However, this method can only be applied to the mechanical systems with constant inertia matrices for the following reason. Since the time derivative of the energy-based Lyapunov function which is used in the passivity-based control depends only on a part of the state of the plant, boundedness of the state can not be guaranteed in the presence of noise which does not vanish at the origin. To solve this problem, we tried to find a Lyapunov function whose time derivative is negative definite, but it was difficult to construct it for general mechanical systems. Consequently, one of the purposes of this paper is to propose a stochastic trajectory tracking control method for general mechanical systems.

Another motivation is that the proposed controllers in [3, 5] are state feedback ones, so full state information is necessary to implement them. However, in practice, there are a lot of mechanical systems whose position measurements are only available because of the lack of velocity sensors. On output feedback control, various methods are proposed, e.g. [6-8]. Observer based control is also studied by many researchers in ,e.g. [9, 10], where the velocity signal is reconstructed by an observer and is utilized for state feedback controller instead of the true signal. On the other hand, in stochastic control field, there are few methods to deal with such a problem. A stochastic output feedback stabilization controller based on the backstepping technique is proposed [11]. However, stochastic trajectory tracking framework by only using position measurements is not considered so far.

This paper proposes an observer based stochastic trajectory tracking control framework. Here we assume that only position measurements can be available. Velocity information is reconstructed by the position information. We consider general mechanical systems as stochastic systems and derive a condition for a stabilizing and tracking controller based on [10, 12, 13] to achieve each control objective in the presence of noise. The controller and observer proposed in [10, 12] utilizes the sliding mode control theory. By taking advantage of those results, we overcome the drawback of our previous result [5] and the proposed method gives conditions for controller and observer gains under which the norm of tracking error remains arbitrarily small in probability.

2. MECHANICAL SYSTEMS IN THE PRESENCE OF NOISE

We consider mechanical systems in the presence of noise as the following stochastic dynamical systems

$$\begin{cases} \mathrm{d}q = v\,\mathrm{d}t + W_q(q,v)\,\mathrm{d}w_1 \\ \mathrm{d}v = M(q)^{-1}\{\tau - C(q,v)v - G(q)\}\,\mathrm{d}t \\ \qquad +W_v(q,v)\,\mathrm{d}w_2 \end{cases} . \quad (1)$$

Here $q(t), v(t) \in \mathbb{R}^n$ describe the generalized coordinate and velocity, respectively. The symmetric positive definite matrix $M(q) \in \mathbb{R}^{n\times n}$ represents the inertia matrix, $C(q,v)v \in \mathbb{R}^n$ represents the Coriolis and centrifugal torques, $G(q) \in \mathbb{R}^n$ denotes the gravitational torques and $\tau \in \mathbb{R}^n$ represents the control input. $w_1(t) \in \mathbb{R}^{r_1}$ and $w_2(t) \in \mathbb{R}^{r_2}$ are standard Wiener processes defined on a probability space $(\Omega, \mathcal{F}, \mathcal{P})$, where Ω is a sample space, $\mathcal{F}$ is the sigma algebra of the observable random events and $\mathcal{P}$ is a probability measure on Ω. $\mathcal{F}_s$ represents the sigma algebra generated by $\{(q(s), v(s)) \mid 0 \le s \le t\}$. $W_q : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n \times r_1}$ and $W_v : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n \times r_2}$ represent noise ports. In the sequel, we define the norm of a matrix A as $\|A\| := \sqrt{\lambda_{\max}(A^\top A)}$, where $\lambda_{\max}(\cdot)$ represents the maximum eigenvalue of $(\cdot)$. $\mathcal{D}_{(\cdot)}$ denotes the derivative with respect to $(\cdot)$. We sometimes treat $M(q)$ as the inertia matrix and other times, as the inertia tensor. For example, for $\alpha, \beta \in \mathbb{R}^n$, $M(q)(\alpha, \beta) := M(q)_{ij}\alpha^i\beta^j \equiv \alpha^\top M(q)\beta$. Here $M(q)_{ij}$, α^i and β^j are the components of $M(q)$, α and β, respectively and we use Einstein summation convention. Tensor notation is a convenient way to describe derivatives. For example, $\mathcal{D}_q M(q)(\alpha, \beta)(\cdot)$ is a first order tensor and is defined as

$$\mathcal{D}_q M(q)(\alpha, \beta)_k = \frac{\partial M(q)_{ij}}{\partial q^k}\alpha^i\beta^j.$$

For a second order tensor $A(\cdot, \cdot)$, we define the transposition $A^\top(\cdot, \cdot)$ as $A^\top_{ij} = A_{ji}$, that is, $A^\top(\alpha, \beta) = A^\top_{ij}\alpha^i\beta^j = A_{ji}\alpha^i\beta^j$.

The matrix $C(q, v)$ has the following property [9, 10].

Remark 1: By defining $C(q, v)$ using the Christoffel symbols, $\mathcal{D}_t M(q) - 2C(q, v)$ is skew-symmetric. Moreover, for this choice $C(q, v)$ satisfies the following equations for any $\xi, \eta, \zeta \in \mathbb{R}^n, a, b \in \mathbb{R}$

$$C(q, \xi)\eta = C(q, \eta)\xi$$

$$C(q, a\zeta + b\xi)\eta = aC(q, \zeta)\eta + bC(q, \xi)\eta.$$

We assume the following.

Assumption 1: W_q and W_v satisfy the local Lipschitz conditions and the linear growth conditions, i.e, for all α, β, there exists positive constants $W_{q,M}$ and $W_{v,M}$ such that

$$\|W_q(\alpha, \beta)\| \le W_{q,M}(1 + \|\alpha\| + \|\beta\|),$$
$$\|W_v(\alpha, \beta)\| \le W_{v,M}(1 + \|\alpha\| + \|\beta\|).$$

Assumption 2: The inertia tensor $M(q)$, its first and second order derivatives with respect to q and the matrix $C(q, \cdot)$ are bounded with respect to q, respectively. For any $q, \xi, \eta \in \mathbb{R}^n$, there exists positive constants satisfying

$$M_m \le \|M(q)\| \le M_M$$
$$\|\mathcal{D}_q M(q)(\xi, \cdot)(\cdot)\| \le \bar{M}_M\|\xi\|$$
$$\|\mathcal{D}^2_q M(q)(\xi, \eta)(\cdot)(\cdot)\| \le \bar{\bar{M}}_M\|\xi\|\|\eta\|$$
$$\|C(q, \xi)\| \le C_M\|\xi\|.$$

3. OBSERVER BASED TRAJECTORY TRACKING CONTROL

In this section, we consider observer based trajectory tracking control of mechanical systems in the presence of noise. Here we assume that only position measurements can be available. Velocity information is reconstructed by the position information.

In the sequel, q_d represents an at least twice differentiable desired trajectory and $\dot{q}_d, \ddot{q}_d$ are the time derivative and the twice time derivative of q_d, respectively. Position and velocity errors are denoted by $q_e := q - q_d, v_e := v - \dot{q}_d$. $\hat{q}$ and $\hat{v}$ denote the estimated position and velocity, respectively and $\tilde{q} := q - \hat{q}, \tilde{v} := v - \hat{v}$ represent the estimation errors. We use the following notation $x_d := (q_d^\top, \dot{q}_d^\top)^\top, x_e := (q_e^\top, v_e^\top)^\top$, $\tilde{x} := (\tilde{q}^\top, \tilde{v}^\top)^\top$ and $x := (x_e^\top, \tilde{x}^\top)^\top$. We assume the following assumption with respect to the desired trajectory.

Assumption 3: the desired trajectory q_d is bounded by N_{q_d} and its velocity $\dot{q}_d$ is bounded by $N_{\dot{q}_d}$, i.e.,

$$N_{q_d} = \sup_t \|q_d(t)\|\ , \quad N_{\dot{q}_d} = \sup_t \|\dot{q}_d(t)\|.$$

We give the notion of (Q_0, Q_1, ρ)-stability [1] in order to consider the stochastic bounded stability.

Definition 1: [1] The systems is (Q_0, Q_1, ρ)-stable if and only if for any initial condition $x(0) \in Q_0$, the probability with respect to the sample path $x(t)$ satisfies

$$\mathcal{P}\{x(t) \in Q_1, \quad \text{for } 0 \le t < \infty\} \ge \rho.$$

The purpose of the section is to derive conditions for the controller and observer gains under which the tracking and estimation error x remains bounded in probability and the margin of error is assignable. The rest of this section, firstly, we define the controller and observer in Eqs. (2) and (3), and then we introduce a stochastic Lyapunov function [1] V in Eq. (6). Secondly, the infinitesimal operator $\mathcal{L}(\cdot)$ is introduced in Eq. (8) in order to calculate the variation of functions along a sample path , since the time variation of the stochastic Lyapunov function plays a key role to investigate the stability as the case of deterministic systems. Then we evaluate a bound of the time variation of V by inequalities in Eq. (15). By utilizing the bound, finally, we prove that a stochastic bounded trajectory tracking control can be achieved if a proposed condition for the controller and observer gains holds.

We utilize the same controller and observer as proposed in [10, 12, 13]. Before defining them, we define the following notations which are used in the literatures

$$s_1 := v_e + \Lambda_1 q_e$$
$$s_2 := \tilde{v} + \Lambda_2\tilde{q},$$
$$v_o := v - s_2 = \hat{v} - \Lambda_2\tilde{q},$$

where Λ_1 and Λ_2 represent positive diagonal matrices, respectively. The controller and observer proposed in [10] are as follows:

$$\tau = M(q)\ddot{q}_d + C(q, v_o)\dot{q}_d + G(q) - K_d(s_1 - s_2), \quad (2)$$

$$\begin{cases} \mathrm{d}\hat{q} = (z + L_d\tilde{q})\,\mathrm{d}t =: \hat{v}\,\mathrm{d}t \\ \mathrm{d}z = L_p\tilde{q}\,\mathrm{d}t + M(q)^{-1}\{\tau - C(q, v_o)\dot{q}_d - G(q) \\ \qquad +K_d(s_1 - s_2)\}\,\mathrm{d}t, \end{cases} \quad (3)$$

where L_d and L_p represent symmetric positive definite matrices, respectively. From Eqs.(1) and (2), the tracking error dynamics is given by

$$
\begin{cases}
\mathrm{d}q_e = v_e\,\mathrm{d}t + W_q(q,p)\,\mathrm{d}w_1 \\
\mathrm{d}v_e = M(q)^{-1}\{C(q,v_o)\dot{q}_d - C(q,v)v - K_d(s_1 - s_2)\}\,\mathrm{d}t \\
\qquad\quad + W_v(q,v)\,\mathrm{d}w_2
\end{cases}
\tag{4}
$$

From Eqs.(1) and (3), the estimation error dynamics is also given by

$$
\begin{cases}
\mathrm{d}\tilde{q} = \tilde{v}\,\mathrm{d}t + W_q(q,p)\,\mathrm{d}w_1 \\
\mathrm{d}\tilde{v} = M(q)^{-1}\{C(q,v_o)\dot{q}_d - C(q,v)v - K_d(s_1 - s_2)\}\,\mathrm{d}t \\
\quad -(L_p\tilde{q} + L_d\tilde{v})\,\mathrm{d}t - L_d W_q(q,v)\,\mathrm{d}w_1 + W_v(q,v)\,\mathrm{d}w_2.
\end{cases}
\tag{5}
$$

We consider the following Lyapunov function proposed in [10] as stochastic Lyapunov function

$$
\begin{aligned}
V &= \frac{1}{2}v_e^\top M(q)v_e + v_e^\top M(q)\Lambda_1 q_e + \frac{1}{2}q_e^\top 2\Lambda_1 K_d q_e \\
&\quad + \frac{1}{2}s_2^\top M(q)s_2 + \frac{1}{2}\tilde{q}^\top 2\Lambda_2 K_d \tilde{q} \\
&= \frac{1}{2}x^\top \underbrace{\begin{bmatrix} 2\Lambda_1 K_d & \Lambda_1 M(q) & 0 & 0 \\ M(q)\Lambda_1 & M(q) & 0 & 0 \\ 0 & 0 & 2\Lambda_2 K_d + \Lambda_2 M(q)\Lambda_2 & \Lambda_2 M(q) \\ 0 & 0 & M(q)\Lambda_2 & M(q) \end{bmatrix}}_{=:\, P(q),} x
\end{aligned}
\tag{6}
$$

where a symmetric positive definite matrix K_d should be chosen so that Schur complement is satisfied, i.e.,

$$
2K_d - M(q)\Lambda_1 > 0.
$$

In order to calculate the time variation of the Lyapunov function (6) along the sample path x governed by Eqs. (4) and (5), we define the infinitesimal operator $\mathcal{L}(\cdot)$.

Definition 2: Consider the nonlinear stochastic differential system written in the sense of Itô

$$
\mathrm{d}x = f(t,x)\,\mathrm{d}t + h(t,x)\,\mathrm{d}w,
\tag{7}
$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$ describe the state and the input, respectively. $w(t) \in \mathbb{R}^r$ is a standard Wiener process. f and h are smooth functions. Then, the infinitesimal generator for the stochastic process of the system (7) is defined as

$$
\mathcal{L}(\cdot) := \mathcal{D}_t(\cdot) + \mathcal{D}_x(\cdot)f + \frac{1}{2}\mathrm{tr}\left\{\mathcal{D}_x^2(\cdot)hh^\top\right\}.
\tag{8}
$$

We can obtain the expectation of the time variation of the Lyapunov function V by calculating $\mathcal{L}(V)$ along the sample path x.

We move on to the calculation of $\mathcal{L}(V)$. From Eqs. (6) and (8), $\mathcal{L}(V)$ is calculated as

$$
\begin{aligned}
\mathcal{L}V =& s_1^\top\{C(q,v_o)\dot{q}_d - C(q,v)v - K_d(s_1 - s_2)\} \\
&+ v_e^\top M(q)\Lambda_1 v_e + \frac{1}{2}s_1^\top \mathcal{D}_t M(q) s_1 \\
&- \frac{1}{2}q_e^\top \Lambda_1 \mathcal{D}_t M(q)\Lambda_1 q_e + 2q_e^\top \Lambda_1 K_d v_e \\
&+ s_2^\top\{C(q,v_o)\dot{q}_d - C(q,v)v - K_d(s_1 - s_2) \\
&- M(q)(L_p\tilde{q} + L_d\tilde{v})\} + s_2^\top M(q)\Lambda_2\tilde{v} \\
&+ \frac{1}{2}s_2^\top \mathcal{D}_t M(q)s_2 + 2\tilde{q}^\top\Lambda_2 K_d\tilde{v} + \omega_{x_e} + \omega_{\tilde{x}},
\end{aligned}
\tag{9}
$$

where ω_{x_e} and $\omega_{\tilde{x}}$ are defined as follows:

$$
\begin{aligned}
\omega_{x_e} &:= \frac{1}{2}\,\mathrm{tr}\left\{\mathcal{D}_{q_e}^2 V W_q W_q^\top\right\} + \frac{1}{2}\,\mathrm{tr}\left\{\mathcal{D}_{v_e}^2 V W_v W_v^\top\right\}, \\
\omega_{\tilde{x}} &:= \frac{1}{2}\,\mathrm{tr}\left\{\mathcal{D}_{\tilde{q}}^2 V W_q W_q^\top\right\} \\
&\quad + \frac{1}{2}\,\mathrm{tr}\left\{\mathcal{D}_{\tilde{v}}^2 V(L_d W_q W_q^\top L_d^\top + W_v W_v^\top)\right\}.
\end{aligned}
\tag{10}
$$

For the simplicity of calculation, we assume the following.

Assumption 4: L_d, L_p, Λ_1 and Λ_2 are constant diagonal matrices and L_d and L_p can be written as

$$
L_d = l_d I + \Lambda_2, \quad L_p = l_d\Lambda_2,
$$

where I represents the identity matrix and a positive constant l_d represents an observer gain.
By utilizing Assumption 4 and Remark 1, we can reduce Eq. (9) to

$$
\begin{aligned}
\mathcal{L}V =& -v_e^\top(K_d - M(q)\Lambda_1)v_e - q_e^\top\Lambda_1 K_d\Lambda_1 q_e - \tilde{v}^\top K_d\tilde{v} \\
&- \tilde{q}^\top\Lambda_2 K_d\Lambda_2\tilde{q} - s_2^\top(l_d M(q) - 2K_d)s_2 \\
&- s_1^\top C(q,s_2)\dot{q}_d + v_e^\top C(q,v)\Lambda_1 q_e + s_2^\top C(q,s_2)v_e \\
&- s_2^\top C(q,v)v_e + \omega_{x_e} + \omega_{\tilde{x}}.
\end{aligned}
\tag{11}
$$

Then we evaluate Equation (11) by utilizing inequalities and Assumptions. In what follows, for a matrix A, A_m and A_M denote lower and upper bounds of $\|A\|$, respectively. Firstly, we consider ω_{x_e} and $\omega_{\tilde{x}}$ in Eq. (10). From $\mathcal{D}_{v_e}^2 V = M(q)$ and Assumptions 1 and 3, we evaluate the second term of ω_{x_e} as

$$
\begin{aligned}
&\mathrm{tr}\left\{\mathcal{D}_{v_e}^2 V W_v W_v^\top\right\} = \sum_{i=1}^{r_2}\lambda_i\left(W_v^\top M W_v\right) \\
&\le r_2\lambda_{\max}\left(W_v^\top M W_v\right) \le r_2\sqrt{M_M}W_{v,M}(1 + \|q\| + \|v\|) \\
&\le r_2\sqrt{M_M}W_{v,M}(1 + \|q_e\| + N_{q_d} + \|v_e\| + N_{\dot{q}_d}).
\end{aligned}
\tag{12}
$$

$$
\begin{aligned}
\mathcal{D}_{q_e}^2 V =& \frac{1}{2}\mathcal{D}_q^2 M(q)(v_e,v_e)(\cdot)(\cdot) + \mathcal{D}_q M(q)(v_e,\Lambda_1(\cdot))(\cdot) \\
&+ \mathcal{D}_q^2 M(q)(v_e,\Lambda_1 q_e)(\cdot)(\cdot) + [\mathcal{D}_q M(q)(v_e)]^\top(\cdot)(\Lambda_1(\cdot)) \\
&+ 2\Lambda_1 K_d
\end{aligned}
$$

and Assumptions 1,2 and 3 yield

$$\operatorname{tr}\left\{\mathcal{D}_{q_e}^2 V W_q W_q^\top\right\} \le r_1\Big(\frac{1}{2}\bar{\bar{M}}_M\|v_e\|^2+2\Lambda_{1,M}\bar{M}_M\|v_e\|+\Lambda_{1,M}\bar{\bar{M}}_M\|q_e\|\|v_e\| + 2K_{d,M}\Lambda_{1,M}\Big)^{\frac{1}{2}}W_{q,M}(1+\|q_e\|+N_{q_d}+\|v_e\|+N_{\dot{q}_d}). \quad (13)$$

Similarly, we have

$$\operatorname{tr}\left\{\mathcal{D}_{\tilde{q}}^2 V W_q W_q^\top\right\} \le \frac{r_1}{2}\Big(\frac{1}{2}\bar{\bar{M}}_M\|s_2\|^2+2\Lambda_{2,M}\bar{M}_M\|s_2\|+\Lambda_{2,M}^2 M_M + 2K_{d,M}\Lambda_{2,M}\Big)^{\frac{1}{2}}W_{q,M}(1+\|q_e\|+N_{q_d}+\|v_e\|+N_{\dot{q}_d})$$

$$\operatorname{tr}\left\{\mathcal{D}_{\tilde{v}}^2 V W_q W_q^\top\right\} \le \frac{\sqrt{M_M}}{2}\left(r_1 L_{d,M}W_{q,M}+r_2 W_{v,M}\right) \times (1+\|q_e\|+N_{q_d}+\|v_e\|+N_{\dot{q}_d}). \quad (14)$$

Consequently, from Eqs. (11),(12),(13) and (14), $\mathcal{L}V$ can be evaluated as

$$\begin{aligned}\mathcal{L}V \le\; & -\min\left\{K_{d,m}-M_M\Lambda_{1,M}, K_{d,m}\Lambda_{1,m}^2, K_{d,m}\Lambda_{2,m}^2\right\}\|x\|^2 \\ & - \left(l_d M_m - 2K_{d,M}\right)\|s_2\|^2 + C_M N_{\dot{q}_d}\|s_1\|\|s_2\| \\ & + C_M(N_{\dot{q}_d}+\|v_e\|)\|v_e\|(\Lambda_{1,M}\|q_e\|+\|s_2\|) \\ & + C_M\|s_2\|^2\|v_e\| + \Big\{\frac{r_1}{2}\Big(\frac{1}{2}\bar{\bar{M}}_M\|v_e\|^2+2\Lambda_{1,M}\bar{M}_M\|v_e\| \\ & + \Lambda_{1,M}\bar{\bar{M}}_M\|q_e\|\|v_e\|+2K_{d,M}\Lambda_{1,M}\Big)^{\frac{1}{2}}W_{q,M} \\ & + \frac{r_2}{2}\sqrt{M_M}W_{v,M}+\frac{r_1}{2}\Big(\frac{1}{2}\bar{\bar{M}}_M\|s_2\|^2+2\Lambda_{2,M}\bar{M}_M\|s_2\| \\ & + \Lambda_{2,M}^2 M_M+2K_{d,M}\Lambda_{2,M}\Big)^{\frac{1}{2}}W_{q,M} \\ & + \frac{\sqrt{M_M}}{2}\Big(r_1(l_d+\Lambda_{2,M})W_{q,M}+r_2W_{v,M}\Big)\Big\} \\ & \times(1+\|q_e\|+N_{q_d}+\|v_e\|+N_{\dot{q}_d}). \end{aligned} \quad (15)$$

Finally, we give a conditions for the controller and observer gains under which the tracking and estimation error remains bounded in probability and the margin of error is assignable. Here we give the following lemma.

Lemma 1: *Consider the system* (1), *the controller* (2), *the observer* (3) *and Assumptions 1,2, 3 and 4. For any* $\delta_0,\ \delta_1 \in \mathbb{R},\ 0<\delta_0<\delta_1$, *we define the following region*

$$T_{\delta_0\delta_1} := \{x \mid \delta_0 \le \|x\| \le \delta_1\}.$$

Then a sufficient condition under which $\mathcal{L}V$ *with respect to the Lyapunov function* V *in* (6) *along the sample path* x *becomes negative is given by*

$$\begin{aligned} & K_{d,m} > \max\Big\{\frac{D(K_d,l_d,\Lambda_1,\Lambda_2,\delta_1)}{\delta_0^2}+M_M\Lambda_{1,M}, \\ & \frac{D(K_d,l_d,\Lambda_1,\Lambda_2,\delta_1)}{\Lambda_{1,m}\delta_0^2}, \frac{D(K_d,l_d,\Lambda_1,\Lambda_2,\delta_1)}{\Lambda_{2,m}\delta_0^2}\Big\}, \\ & l_d M_m > 2K_{d,M}, \end{aligned} \quad (16)$$

where $D(K_d,l_d,\Lambda_1,\Lambda_2,\delta_1)$ *is defined as*

$$\begin{aligned} D(K_d,l_d,\Lambda_1,\Lambda_2,\delta_1) :=\; & C_M(\Lambda_{1,M}+2)\delta_1^2(N_{\dot{q}_d}+\delta_1) \\ & + \Big\{\frac{r_1}{2}\Big(\bar{\bar{M}}_M\delta_1^2\Big(\frac{1}{2}+\Lambda_{1,M}\Big)+2\Lambda_{1,M}\bar{M}_M\delta_1 \\ & + 2K_{d,M}\Lambda_{1,M}\Big)^{\frac{1}{2}}W_{q,M}+\frac{r_2}{2}\sqrt{M_M}W_{v,M} \\ & + \frac{r_1}{2}\Big(\frac{1}{2}\bar{\bar{M}}_M(1+\Lambda_{2,M})^2\delta_1^2+2\Lambda_{2,M}\bar{M}_M(1+\Lambda_{2,M})\delta_1 \\ & + \Lambda_{2,M}^2M_M+2K_{d,M}\Lambda_{2,M}\Big)^{\frac{1}{2}}W_{q,M} \\ & + \frac{\sqrt{M_M}}{2}\Big(r_1(l_d+\Lambda_{2,M})W_{q,M}+r_2W_{v,M}\Big)\Big\} \\ & \times(1+N_{q_d}+N_{\dot{q}_d}+2\delta_1). \end{aligned} \quad (17)$$

Proof: Under the condition $l_dM_m > 2K_{d,M}$, Eq. (15) is reduced to the following on $T_{\delta_0\delta_1}$

$$\begin{aligned}\mathcal{L}V \le\; & -\min\left\{K_{d,m}-M_M\Lambda_{1,M}, K_{d,m}\Lambda_{1,m}^2, K_{d,m}\Lambda_{2,m}^2\right\}\delta_0^2 \\ & + D(K_d,l_d,\Lambda,\delta_1). \end{aligned} \quad (18)$$

The rest of the condition in (16) is immediately derived from Eq. (18). ■

We are ready to state the main result.

Theorem 1: Consider the system (1), the controller (2), the observer (3) and Assumptions 1,2, 3 and 4. For any $\lambda_1 \in \mathbb{R},\ \lambda_1>0$ and $\rho\in\mathbb{R},\ 0<\rho<1$, we assign any λ_0 which satisfies $0<\lambda_0<\lambda_1(1-\rho)$. Then, under the condition (16) in Lemma 1 with the following δ_0 and δ_1:

$$\delta_0 = \sqrt{\frac{2\lambda_0}{P_M}}, \quad \delta_1 = \sqrt{\frac{2\lambda_1}{P_m}}, \quad (19)$$

the system is $(Q_0, R_{\lambda_1}, \rho)$-stable, where Q_0 and R_{λ_1} are defined as

$$\begin{aligned} Q_0 &:= \{x \mid \lambda_0 < V(x) < \lambda_1(1-\rho)\} \\ R_{\lambda_1} &:= \{x \mid V(x)<\lambda_1\}. \end{aligned} \quad (20)$$

In Eq. (19), P_m and P_M represent lower and upper bounds for $P(q)$ in (6), i.e, $P_m \le \|P(q)\| \le P_M,$ for $^\forall q$ holds.

Under the condition, the following probability inequality is also holds

$$\mathcal{P}\left\{\sup_{0\le t<\infty}\|x(t)\| < \sqrt{\frac{2\lambda_1}{P_M}}\right\} \ge \rho.$$

Before proving the theorem, the stopped process [1] is introduced.

Definition 3: [1] Define $t\cap s := \min\{t,s\}$. Suppose that τ_S is the first time of exit of the process $x(s)$ from an open set S, i.e., $\tau_S := \inf\{t\ge 0 \mid x(t)\ne S\}$. Then, the stopped process $x(t\cap\tau_S)$ is defined as

$$x(t\cap\tau_S) := \begin{cases} x(t) & t<\tau_S \\ x(\tau_S) & t\ge\tau_S \end{cases}.$$

Here we prove the Theorem 1.

Proof: Firstly, we derive the condition under which $\mathcal{L}V(x)$ is negative on the region of x satisfying $\lambda_0 < V(x) < \lambda_1$. From

$$\frac{1}{2}P_m\|x\|^2 \le V(x) \le \frac{1}{2}P_M\|x\|^2, \tag{21}$$

it is sufficient to consider the condition under which

$$\mathcal{L}V(x) < 0 \quad \text{on} \quad \sqrt{\frac{2\lambda_0}{P_M}} < \|x\| < \sqrt{\frac{2\lambda_1}{P_m}}. \tag{22}$$

By utilizing Lemma 1, a condition satisfies (22) is easily obtained as the condition (16) with δ_0 and δ_1 defined in Eq. (19). With the condition and the Dynkin's formula, for $0 \le s \le t$ we have

$$\begin{aligned} &E[V(x(t\cap\tau_{S_{\delta_0\delta_1}}))] - V(x(s)) \\ &\quad = E\left[\int_s^{t\cap\tau_{S_{\delta_0\delta_1}}}\mathcal{L}V(x(u))\,\mathrm{d}u\right] < 0. \end{aligned} \tag{23}$$

Since $E[V(x(t\cap\tau_{S_{\delta_0\delta_1}}))|\mathcal{F}_s] < V(x(s))$ holds from Eq. (23), $\{V(x(t\cap\tau_S));\ t\ge 0\}$ is a nonnegative supermartingale. By utilizing the supermartingale property [14, 15], we obtain the following probability inequality

$$\begin{aligned} \frac{V(x(0))}{\lambda_1} &\ge \mathcal{P}\left\{\sup_{0\le t<\infty} V(x(t\cap\tau_{S_{\delta_0\delta_1}})) \ge \lambda_1\right\} \\ &= \mathcal{P}\left\{\sup_{0\le t<\infty} V(x(t)) \ge \lambda_1\right\}. \end{aligned}$$

Consequently, if the initial condition for $x(0)$ is chosen from Q_0 in (20), we have

$$\mathcal{P}\left\{\sup_{0\le t<\infty} V(x(t)) < \lambda_1\right\} \ge 1 - \frac{\lambda_1(1-\rho)}{\lambda_1} = \rho. \tag{24}$$

Under the condition which Eq.(24) holds, the last statement can be easily shown, since Eq. (21) holds.

This proves the theorem. ■

4. CONCLUSION

This paper has introduced an observer based stochastic trajectory tracking control of mechanical systems. We considered mechanical systems in the presence of noise as stochastic systems and the situation that only position measurements was available. We have derived conditions for the controller and observer gains under which the tracking and estimation error remains bounded in probability and the margin of error is assignable. The stochastic bounded stability has been proven by utilizing supermartingale property.

We will investigate the existence of parameters which always hold the proposed condition and a systematic method for controller design.

REFERENCES

[1] H. J. Kushner, *Stochastic Stability and Control*. Academic Press, 1967.

[2] P. Florchinger, "A passive system approach to feedback stabilization of nonlinear control stochastic systems," *SIAM J. Control Optim.*, vol. 37, no. 6, pp. 1848–1864, 1999.

[3] S. Satoh and K. Fujimoto, "On passivity based control of stochastic port-hamiltonian systems," in *Proc. 47th IEEE Conf. on Decision and Control*, 2008, pp. 4951–4956.

[4] B. Maschke and A. J. van der Schaft, "Port-controlled Hamiltonian systems: modelling origins and system theoretic properties," in *Proc. 2nd IFAC Symp. Nonlinear Control Systems*, 1992, pp. 282–288.

[5] S. Satoh and K. Fujimoto, "Stabilization of time-varying stochastic port-Hamiltonian systems and its application to stochastic trajectory tracking control," 2008, pp. 393–398, proc. 37th SICE Symposium on Control Theory (in Japanese).

[6] M. Takegaki and S. Arimoto, "A new feedback method for dynamic control of manipulators," *Trans. ASME, J. Dyn. Syst., Meas., Control*, vol. 103, pp. 119–125, 1981.

[7] S. Stramigioli, B. M. J. Maschke, and A. J. van der Schaft, "Passive output feedback and port interconnection," in *Proc. 4th IFAC Symp. Nonlinear Control Systems*, 1998, pp. 613–618.

[8] R. Ortega, A. Loría, R. Kelly, and L. Praly, "On passivity-based output feedback global stabilization of euler-lagrange systems," in *Proc. 33rd IEEE Conf. on Decision and Control*, 1994, pp. 381–386.

[9] S. Nicosia and P. Tomei, "Robot control by using only joint position measurements," *IEEE Trans. Autom. Contr.*, vol. 35, no. 9, pp. 1058–1061, 1990.

[10] H. Berghuis and H. Nijmeijer, "A passivity approach to controller-observer design for robots," *IEEE Trans. Robotics and Automation*, vol. 9, no. 6, pp. 740–754, 1993.

[11] H. Deng and M. Krstić, "Output-feedback stochastic nonlinear stabilization," *IEEE Trans. Autom. Contr.*, vol. 44, no. 2, pp. 328–333, 1999.

[12] J.-J. E. Slotine and W. Li, "On the adaptive control of robot manipulators," *Int. J. Robotics Research*, vol. 6, no. 3, pp. 49–59, 1987.

[13] B. Paden and R. Panja, "Globally asymptotically stable 'pd+' controller for robot manipulators," *Int. J. Control*, vol. 47, no. 6, pp. 1697–1712, 1988.

[14] E. B. Dynkin, *Markov Processes*. Berlin: Springer, 1965.

[15] H. J. Kushner, *Stochastic stability*, ser. Stability of Stochastic Dynamical Systems; Lecture Notes in Math. 294, R. Curtain, Ed. Berlin and New York: Springer-Verlag, 1972.